

# A Task Model for TDMA Communications: Application to Software Radio Protocols

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**Abstract**—In this article a new task model is proposed for scheduling analysis of dependent tasks in radio stations that embed a TDMA communication protocol. TDMA is a channel access protocol that allows several stations to communicate in a same network, by dividing time into several time slots. Tasks handling the TDMA radio protocol are scheduled in a manner to be compliant with the TDMA configuration: task parameters such as execution times, deadlines and release times are constrained by TDMA slots. The periodic task model, commonly used in scheduling analysis, is inefficient for the accurate specification of such systems, resulting in pessimistic scheduling analysis results. To encompass this issue, this article proposes a new task model called Dependent General Multiframe (DGMF). This model extends the existing GMF model with precedence dependency and shared resource synchronization. We show how to perform scheduling analysis with DGMF by transforming it into a transaction model and using a schedulability test we proposed. In this article we experiment on "software radio protocols" from Thales Communications & Security, which are representative of the system we want to analyze. Experimental results show an improvement of system schedulability using the proposed analysis technique, compared to existing ones (GMF and periodic tasks). The new task model thus provides a technique to model and analyze TDMA systems with less pessimistic results.

## I. INTRODUCTION

This article proposes to improve scheduling analysis of systems with Time Division Multiple Access (TDMA) [2] communications. In this kind of system, the TDMA communication protocol has an impact on task semantics which leads to pessimistic scheduling analysis with the periodic task model [16]. Furthermore, we analyze systems where tasks have dependencies, which, if not considered, may lead to wrong scheduling analysis results. To specify more accurately tasks in such systems, a new task model is proposed: the Dependent General Multiframe (DGMF) model, which extends the General Multiframe (GMF) [1] model with task dependencies. This task model is applied to TDMA Software Radio Protocols (SRP) [19], systems for which the mentioned analysis issues apply.

An SRP is a software implementing a communication protocol embedded in radio stations that are part of a (mobile ad-hoc) wireless network. It is a real-time software that runs on an execution platform (e.g. operating system and hardware) and it controls how the radio's physical equipment behaves in terms of data transmission/reception over the air. An SRP is a multi-tasked system with time-constrained tasks

accessing the execution platform's processors according to a scheduling policy. Tasks may have dependencies through precedence dependencies (e.g. communications) and shared resource synchronizations.

TDMA is a channel access protocol, in which a TDMA frame is synchronized between all stations. The frame is divided into several contiguous time slots of different types and durations. When critical tasks are released by slots, task scheduling depends on the way the TDMA frame is configured. As an example, a task released by a slot has its execution time that depends on the slot's type, its deadline that depends on the slot's duration, and its release time that depends on the slot's start time.

Scheduling analysis [23] is a typical technique used to verify that tasks meet their time constraints. Scheduling analysis techniques assume that simplified task models are used to model the analyzed system. The periodic task model [16] is an example of such simplified task model. In [15], we modeled and performed scheduling analysis of a TDMA SRP system with the periodic model. It was shown that this model leads to pessimistic scheduling analysis results in this context.

In this article a new task model is proposed, to increase accuracy of scheduling analysis of TDMA oriented systems. Our model is based on the GMF model. This model was proposed to improve scheduling analysis of video decoders/encoders where task parameters depend on incoming video frames. A TDMA SRP's behavior is similar to this kind of application. Unfortunately GMF does not allow to specify task dependencies (i.e. precedence and shared resource) and SRPs have such kind of constraint. We thus propose a new GMF model called DGMF, to extend GMF with tasks dependencies.

The rest of the article is organized as follows: in section II, we present the TDMA SRP system, i.e. our system scope and assumptions. In section III the DGMF task model is proposed. Section IV shows how to perform scheduling analysis of DGMF tasks by transforming them to transactions. In section V, experimental results show the DGMF to transaction transformation's correctness, performance, and how it is applied to a TDMA SRP from Thales. In section VI this article's approach is compared to related work. We conclude and discuss future work in section VII.

## II. SOFTWARE RADIO PROTOCOL

In this article, we consider SRP embedded in a radio station which is part of a larger physical system (e.g. helicopter, vehicular transport). An SRP is a software that implements a communication protocol. Most of the time, these radio stations communicate in a mobile ad-hoc wireless network. An SRP observes events occurring in the network (e.g. stations appearing, disappearing), transmits/receives messages and reroutes them to other stations when necessary. When a message is received, the SRP can send it to a user system or an equipment that may control the physical system in which the SRP is embedded. We assume that the effects of non-determinism in wireless networks, on scheduling analysis of a single station, are negligible.

TDMA is a common communication protocol in SRPs. In the following sections, we present a TDMA SRP through its system view and then its software and execution platform view.

### A. System View

From a system point of view, a SRP is divided into several layers. Fig. 1 shows an example of such layers.

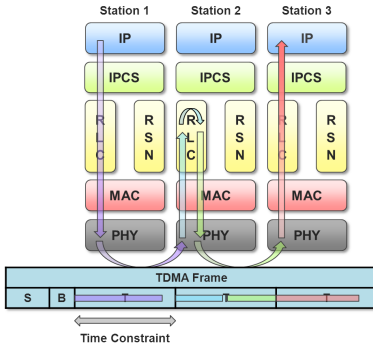


Fig. 1. TDMA SRP System View Example

In Fig. 1, the *IPCS* layer interfaces with the *IP* stack of the user system above. The *RLC* layer handles translation between IP packets and radio protocol packets. It also reroutes incoming packets if necessary (e.g. a received packet's destination is a neighbor). The *RSN* layer handles network topology and address updates (e.g. address of neighbor stations in the network appearing/disappearing). When an SRP uses TDMA, the *MAC* layer handles the TDMA protocol by preparing/receiving protocol packets for/from the *PHY* layer that sends them over the air.

In Fig. 1, control and data flows pass through the different layers. The flows are constrained by the TDMA frame. A TDMA frame is divided into several time slots of different types, durations, and modes. For example in Fig. 1, the TDMA frame has three kinds of slot: *Service* (*S*) for synchronization between stations; *Broadcast* (*B*) for observation/signaling of/on the network; *Traffic* (*T*) for effective data transmission/reception. Slots of different types do not have the same duration (e.g. a *B* slot is shorter than a *S* and *T* slot). Slots can either be in *Tx* (transmission), *Rx* (reception), or *Idle* mode.

A TDMA configuration defines the combination of slots (type and mode) in a TDMA frame. A TDMA frame is repeated after it finishes, with possibly a different configuration. We assume that in a TDMA configuration, only the slot modes change from one TDMA frame to the next.

### B. Software and Execution Platform

Fig. 2 shows an example of the software and execution platform architecture of an SRP.

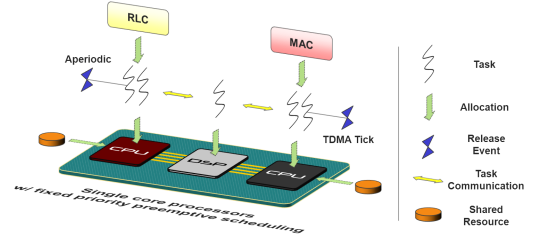


Fig. 2. Software and Execution Platform Architecture Example

From Fig. 2, we see that the layers are implemented by tasks allocated on processors. The tasks thus handle the flows that pass through layers. Tasks are scheduled by a **fixed priority preemptive policy**. Tasks may have **precedence dependency** and used **shared resources** protected by a protocol [24]. Tasks that handle the TDMA protocol have **hard deadlines** and are constrained by the TDMA frame. For example a task must be released by a TDMA "tick" indicating the start of a slot; a task has an execution time that depends on a specific slot; and a task must finish before some next slot. These tasks are thus the ones that interest us for scheduling analysis

## III. DEPENDENT GENERAL MULTIFRAME

Dependent General Multiframe (DGMF) is a GMF task model extended with task precedence dependency and shared resource synchronization.

GMF is well suited for TDMA SRPs where task parameters depend on TDMA slots. Indeed a GMF task is a vector of frames (not to be confused with "TDMA frame") with different parameters. This task model suits tasks released by slots of a TDMA frame, having different task parameters at each slot. However, we have to extend GMF with task dependencies because TDMA SRPs have such requirements.

In the following sections, a DGMF task is defined. An example of modeling a TDMA SRP's tasks with DGMF is then shown. We finish by discussing about scheduling analysis for DGMF.

### A. DGMF Definitions

A DGMF task  $G_i$  is a vector composed of  $N_i$  frames  $F_i^j$ , with  $1 \leq j \leq N_i$ . Each frame has parameters:

- $E_i^j$  [1] is the Worst Case Execution Time (WCET) of  $F_i^j$ .
- $D_i^j$  [1] is the relative deadline of  $F_i^j$ .
- $P_i^j$  [1] is min-separation of  $F_i^j$ , i.e. the minimum time separating the release of  $F_i^j$  and the release of  $F_i^{j+1}$ .

- $[U]_i^j$  is a set of  $(R, S, B)$  tuples denoting shared resource critical sections.  $F_i^j$  allocates resource  $R$  after it has run  $S$  time units of its execution time, and then locks the resource during the next  $B$  time units of its execution time.
- $[F_p^q]_i^j$  is a set of predecessor frames, i.e. frames from any other DGMF task that must finish before  $F_i^j$  can be released. A predecessor frame is denoted  $F_p^q$ . To avoid communication buffer overflows,  $F_p^q$  can be in  $[F_p^q]_i^j$  only if  $G_i$  and  $G_p$  have the same *DGMF-Period* (defined below).
- $prio(F_i^j)$  is the priority of  $F_i^j$ .
- $proc(F_i^j)$  is the processor on which  $F_i^j$  is allocated on.

For GMF task  $G_i$  with  $N_i$  frames, we define the *DGMF-Period* of  $G_i$  as:  $T_{G_i} = \sum_{j=1}^{N_i} P_i^j$

Frames are released cyclically [1]: frames are released in the order defined by the vector and any released frame  $F_i^{j+k \times N_i}$  ( $k > 0$ ) has parameters equal to frame  $F_i^j$ . The first frame to be released by a DGMF task  $G_i$  is always the first frame in its vector, denoted  $F_i^1$ .

In the GMF model, time between events was only set between frames of a same task, through the  $P_i^j$  parameter. We introduce another parameter to allow further capturing of time between events. For example a task can be specified to be released for the first time only two slots after the beginning of the TDMA frame. We call  $r_i^1$ , the release time of first frame  $F_i^1$ . We call  $r_i$ , the release time of  $G_i$  and we thus have  $r_i = r_i^1$ . For any  $F_i^j$ , with  $j > 1$ , their release time is:  $r_i^j = r_i^1 + \sum_{h=1}^{j-1} P_i^h$ . Finally for any  $F_i^j$ , we call value  $r_i^j + D_i^j$  the global deadline of  $F_i^j$ .

A DGMF task set may have the following property:

*Property 1 (Unique Predecessor):* Let  $F_i^j$  be a frame of a task  $G_i$ , in a DGMF task set. Let  $[F_p^q]_i^j$  be the set of predecessor frames of  $F_i^j$ .  $F_i^{j-1}$  is the previous frame of  $F_i^j$  in the vector of  $G_i$ , if  $j > 1$ . The set of frames that precede  $F_i^j$  is the set  $[F_p^q]_i^j$  and  $F_i^{j-1}$  (if  $j > 1$ ). A DGMF task set is said to respect the *Unique Predecessor* property if, for all frames  $F_i^j$ , there is at most one frame  $F_x^y$ , among frames that precede  $F_i^j$ , with a global deadline (i.e.  $r_x^y + D_x^y$ ) greater than or equal to the release time of  $F_i^j$ . Formally the *Unique Predecessor* property is defined as:

$$\exists_{\leq 1} F_x^y \in pred(F_i^j), r_x^y + d_x^y \geq \max_{F_l^h \in pred(F_i^j)} (r_l^h + E_l^h), r_i^j \quad (1)$$

Where  $\exists_{\leq 1}$  means "there exist at most one", and the set  $pred(F_i^j)$  is defined as:

$$pred(F_i^j) = \begin{cases} [F_p^q]_i^j \cup \{F_i^{j-1}\} & \text{if } j > 1 \\ [F_p^q]_i^j & \text{otherwise.} \end{cases} \quad (2)$$

We assume that TDMA SRPs are modeled with DGMF task sets having the *Unique Predecessor* property.

## B. DGMF Example

Consider the DGMF task set in Table I, modeling tasks constrained by a TDMA frame.

TABLE I  
DGMF TASK SET

	$E_i^j$	$D_i^j$	$P_i^j$	$[U]_i^j$	$[F_p^q]_i^j$
$G_1; r_1 = 0$					
$F_1^1$	1	4	1		$F_2^1$
$F_1^2$	1	3	1		
$F_1^3$	1	2	6		
$F_1^4$	1	4	4		$F_2^2$
$F_1^5$	4	8	8	(R, 1, 3)	$F_2^3$
$G_2; r_2 = 0$					
$F_2^1$	1	4	8		$F_{Tick}$
$F_2^2$	1	4	4		
$F_2^3$	1	4	4		
$F_2^4$	2	4	4	(R, 0, 1)	
$G_3; r_3 = 4$					
$F_3^1$	1	2	2		$F_4^1$
$F_3^2$	1	2	18		$F_4^2$
$G_4; r_4 = 4$					
$F_4^1$	1	2	2		$F_{Tick}$
$F_4^2$	1	2	18		
$Tick; r_{Tick} = 0$					
$F_{Tick}$	0	$+\infty$	20		

Frames of  $G_1$  and  $G_3$  have a priority of 1. Frames of  $G_2$  and  $G_4$  have a priority of 2. All frames are allocated on *CPU1* except  $F_1^2$ , which is allocated on *CPU2*, and  $F_{Tick}$ , which is allocated on *CPU3*. Task *Tick* is similar to the idea of the "ghost root task" introduced in [22]. Its only purpose is to represent the first TDMA tick that starts the whole TDMA frame. Task *Tick* ensures that all tasks constrained by the TDMA frame are part of the same precedence dependency graph. Fig. 3 shows an example of a schedule produced by the task set, over 20 time units. In Fig. 3, there is a TDMA frame of 1 *S* slot, 2 *B* slots, and 3 *T* slots.  $G_2$  is released at *S* and *T* slots.  $G_2$  releases  $G_1$  upon completion.  $G_4$  is released at *B* slots.  $G_4$  releases  $G_3$  upon completion. The release time parameter  $r_i$  allows to specify the time between releases of  $G_2$ ,  $G_4$ , and the TDMA slots.

## C. Applicability of GMF Scheduling Analysis on DGMF

Scheduling analysis techniques exist for independent GMF tasks. Let us see if they can be applied to DGMF tasks.

In [1] a processor utilization based feasibility test for GMF tasks is proposed. The test runs in polynomial time and is restricted to independent tasks running on a uniprocessor system under a preemptive Earliest Deadline First (EDF) scheduling policy. In [25] a response time based schedulability test for GMF tasks is proposed. This test assumes that tasks are independent and run on a uniprocessor system with a preemptive FP scheduling policy. Furthermore the authors also assume the relative deadline of a frame to be smaller than the release time of the next frame.

Obviously original analysis techniques for GMF tasks cannot be directly applied to DGMF tasks due to task dependencies. For example in the original tests [1], [25], frames can be

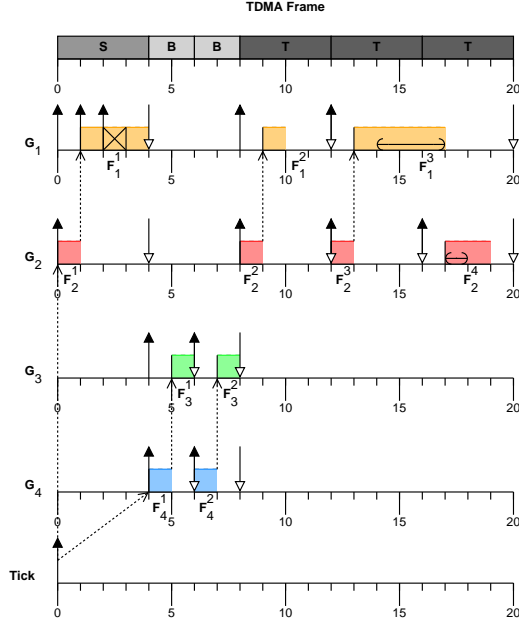


Fig. 3. DGMF Tasks in a TDMA SRP: Up arrows are frame releases; Down arrows are frame relative deadlines; Dashed arrows are precedence dependencies; Curved arrows are shared resource allocations; Crossed frame executes on different processor

released simultaneously as long as they belong to different GMF tasks. This no longer holds true when precedence dependencies are defined between frames.

The following section shows how the transaction task model is used to analyze schedulability of DGMF tasks.

#### IV. DGMF SCHEDULING ANALYSIS USING TRANSACTIONS

The transaction model [27] was originally proposed to model distributed systems and compute end-to-end response times. This model allows specification of precedence dependency and shared resource synchronization.

In [21] a GMF to transaction transformation algorithm is proposed. Authors in [21] argue that scheduling analysis techniques for transactions can be applied to GMF tasks. We propose to use transactions, and their associated scheduling analysis techniques, to perform scheduling analysis of DGMF tasks. The transformation algorithm in [21] is thus extended for DGMF.

In the following sections the transaction model is defined. We then show how to transform DGMF tasks to transactions. Afterwards, a typical transformation example is presented. From this example we choose a scheduling analysis technique applicable to transactions resulting from the transformation.

##### A. Transaction Definitions

From [20], a transaction (denoted  $\Gamma_i$ ) is a group of tasks (denoted  $\tau_{ij}$ ). A transaction is released by a periodic event that occurs every  $T_i$ . A particular instance of a transaction is

called a job. In this article, we call  $r_i$  the release time of a transaction  $\Gamma_i$ , i.e. of the first job of  $\Gamma_i$ .

A job of a task in a transaction is released after/by the event that releases the job of the transaction. If the event that releases the  $p^{th}$  job of  $\Gamma_i$  occurs at  $t_0$ , then the  $p^{th}$  jobs of its tasks are released after/at  $t_0$ . Each task has parameters:

- $C_i^j$  is the WCET. We assume the Best Case Execution Time (BCET) of a task is equal to its WCET.
- $O_i^j$  is the offset, i.e.  $\tau_i^j$  is released at least  $O_i^j$  units of time after  $t_0$ . Value  $r_i^j = r_i + O_i^j$  is called the release time of  $\tau_i^j$ .
- $d_i^j$  is the relative deadline, i.e. the response time of  $\tau_i^j$  must be smaller than  $O_i^j + d_i^j$ . Value  $O_i^j + d_i^j$  is called the global deadline [20] of  $\tau_i^j$ . Value  $t_0 + O_i^j + d_i^j$  is called the absolute deadline of a job of  $\tau_i^j$ .
- $J_i^j$  is the maximum jitter, i.e.  $\tau_i^j$  is released in  $[t_0 + O_i^j; t_0 + O_i^j + J_i^j]$ .
- $B_i^j$  is the maximum shared resource blocking time [24].
- $prio(\tau_i^j)$  is the priority.
- $proc(\tau_i^j)$  is the processor on which  $\tau_i^j$  is allocated on.

Tasks in a transaction are related by precedence dependencies [20]. A precedence dependency between two tasks, denoted  $\tau_{ip} \prec \tau_{ij}$ , is a constraint that means that a job  $p$  of  $\tau_{ip}$  must finish before a job  $p$  of  $\tau_{ij}$  can be released.  $\tau_{ip}$  (resp.  $\tau_{ij}$ ) is called the predecessor (resp. successor) of  $\tau_{ij}$  (resp.  $\tau_{ip}$ ).

Tasks may access shared resources in critical sections [24]. In this article, a critical section is denoted  $(\tau, R, S, B)$  where  $\tau$  is the task accessing the resource  $R$ ,  $S$  is the resource allocation time, and  $B$  is the resource blocking time.

##### B. DGMF To Transaction

The DGMF to transaction transformation aims at expressing parameters and dependencies in the DGMF model as ones in the transaction model. The transformation has three major steps:

- 1 Independent DGMF to Transaction: Consider DGMF tasks independent and transform to transactions.
- 2 Add Shared Resource Synchronizations: Express critical sections in the resulting transaction set.
- 3 Add Precedence Dependencies: Model precedence dependencies in transaction model [20].

In the following sections each step is explained in detail. We will also see that the transformation already starts assessing schedulability.

1) *Independent DGMF to Transaction*: **Step 1** consists in transforming each DGMF task to a transaction by considering DGMF tasks as independent. Algorithm IV-B1.1 shows the original algorithm proposed by [21], that we extend for DGMF tasks. The idea behind the algorithm is to transform frames  $F_i^j$  of a DGMF task  $G_i$  into tasks  $\tau_i^j$  of a transaction  $\Gamma_i$ . Parameters in the transaction model, like WCET ( $C_i^j$ ), relative deadline ( $d_i^j$ ) and priority ( $prio(\tau_i^j)$ ), are computed from parameters  $E_i^j$ ,  $D_i^j$  and  $prio(F_i^j)$  from the DGMF model. To transform the min-separation between two frames in the

DGMF model, offsets ( $O_i^j$ ) are used in the transaction model. The offset of a task  $\tau_i^j$  is computed by summing the  $P_i^h$  of frames  $F_i^h$  preceding  $F_i^j$  in the vector of  $G_i$ . In our extension of the transformation, the release time  $r_i$  of a DGMF task  $G_i$  is transformed into the release time  $r_i$  of a transaction  $\Gamma_i$ .

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**Algorithm IV-B1.1** Independent DGMF to Transaction

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```

1: for each DGMF task  $G_i$  do
2:   Create transaction  $\Gamma_i$ 
3:
4:    $T_i \leftarrow \sum_{j=1}^{N_i} P_i^j$ 
5:    $\Gamma_i.r_i \leftarrow G_i.r_i$ 
6:
7:   for each  $F_i^j$  in  $G_i$  do
8:     Create task  $\tau_i^j$  in  $\Gamma_i$ 
9:
10:     $C_i^j \leftarrow E_i^j$ 
11:     $d_i^j \leftarrow D_i^j$ 
12:     $J_i^j \leftarrow 0$ 
13:     $B_i^j \leftarrow 0$ 
14:     $prio(\tau_i^j) \leftarrow prio(F_i^j)$ 
15:     $proc(\tau_i^j) \leftarrow proc(F_i^j)$ 
16:    if  $j = 1$  then
17:       $O_i^j \leftarrow 0$ 
18:    else
19:       $O_i^j \leftarrow \sum_{h=1}^{j-1} P_i^h$ 
20:    end if
21:  end for
22: end for

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*Proof of Algorithm IV-B1.1:* Algorithm IV-B1.1 in is based on the algorithm in [21]. The algorithm is proven by construction. ■

2) *Add Shared Resource Synchronizations:* In **Step 2**, if a critical section is defined for a frame, then the task, corresponding to the frame after transformation, must also define the critical section. If there is a critical section  $(R, S, B)$  in  $[U]_i^j$ , then a critical section  $(\tau_i^j, R, S, B)$  must be specified in the transaction set resulting from **Step 1**. When all frame critical sections have been transformed,  $B_i^j$  of each  $\tau_i^j$  is computed [24].

*Proof of Step 2:* Task  $\tau_i^j$  is the result of the transformation of  $F_i^j$ , thus by construction we must have  $(\tau_i^j, R, S, B)$  if we have  $(R, S, B) \in [U]_i^j$ . ■

3) *Add Precedence Dependencies:* The goal of **Step 3** is to add precedence dependencies to the transaction set, with respect to how they are modeled in the transaction model (i.e. with offsets and jitters [20]). **Step 3** is divided into three sub-steps:

- 3.a** Express Precedence Dependency Constraints: In the transaction set, express precedence dependency constraints from the DGMF set.
- 3.b** Model Precedence Dependency in the Transaction Model: Modifications of releases and offsets so precedence dependencies in the transaction set are modeled according to [20].
- 3.c** Reduce Precedence Dependencies: Simplify the transaction set by reducing number of precedence dependency constraints.

The following paragraphs present each of these sub-steps.

a) *Express Precedence Dependency Constraints:* We define two kinds of precedence dependency in the DGMF model: intra and inter. We call intra dependency a precedence dependency that is implicitly expressed between frames of a same DGMF task. Frames of a DGMF task execute in the order defined by the vector. An inter dependency is a precedence dependency between frames belonging to different DGMF tasks.

An intra dependency in the DGMF set is expressed in the transaction set by adding a precedence dependency between tasks, representing successive frames, if they are part of a same transaction resulting from **Step 1** (i.e.  $\tau_i^j \prec \tau_i^{j+1}$  with  $j < N_i$ ). This also ensures that these tasks are part of the same precedence dependency graph, which is important for determining the transaction's characteristics, as we will see later. Inter dependencies must also be expressed in the transaction set resulting from **Step 1**. If a frame  $F_p^q$  of task  $\tau_p^q$  is in the set of predecessor frames  $[F_p^q]_i^j$  of task  $\tau_i^j$ , then a precedence dependency  $\tau_p^q \prec \tau_i^j$  is added.

*Proof of Step 3.a:* By definition frames of  $G_i$  are released in the order defined by the vector of  $G_i$  so  $F_i^j$  precedes  $F_i^{j+1}$  ( $j < N_i$ ). Task  $\tau_i^j$  (resp.  $\tau_i^{j+1}$ ) is the result of the transformation of  $F_i^j$  (resp.  $F_i^{j+1}$ ), thus by construction we must have  $\tau_i^j \prec \tau_i^{j+1}$ . The same proof is given for  $\tau_p^q \prec \tau_i^j$ , resulting from the transformation of  $F_p^q \in [F_p^q]_i^j$ . ■

b) *Model Precedence Dependency in the Transaction Model:* In the transaction model, precedence dependencies should be modeled with the method in [20]. This is done in **Step 3.b** with three algorithms: **(ALG1)** Task Release Time Modification; **(ALG2)** Transaction Merge; and **(ALG3)** Transaction Release Time Modification.

**(ALG1) Task Release Time Modification**

In **ALG1** the release time  $r_i^j$  of each task  $\tau_i^j$ , in the transaction set, is modified according to precedence dependencies. This enforces that the release time of  $\tau_i^j$  is larger than the latest completion time ( $r_p^q + C_p^q$ ) of a predecessor  $\tau_p^q$  of  $\tau_i^j$ . Task release times are changed by modifying offsets because  $r_i^j = r_i + O_i^j$ , where  $r_i$  is the release time of  $\Gamma_i$ . The release time modification algorithm is shown in Algorithm IV-B3.1. Since the release time of  $\tau_p^q$  may also be modified when the algorithm runs, release time modifications are made until no more of them occur. Note that when the offset  $O_i^j$  of  $\tau_i^j$  is increased, its relative deadline ( $d_i^j$ ) is shortened and then compared to its WCET ( $C_i^j$ ) to verify if the relative deadline is sure to be missed.

*Proof of Algorithm IV-B3.1:* Let us assume  $\tau_p^q \prec \tau_i^j$ . The earliest release time of  $\tau_i^j$  is  $r_i^j$ . We remind that it is assumed that the BCET of  $\tau_p^q$  is equal to its WCET. According to [3],  $\tau_p^q \prec \tau_i^j \Rightarrow r_p^q + C_p^q \leq r_i^j$  is true. The implication is false only if  $\neg(r_p^q + C_p^q \leq r_i^j) \Leftrightarrow r_p^q + C_p^q > r_i^j$ . Therefore if we have  $\tau_p^q \prec \tau_i^j$  then we cannot have  $r_p^q + C_p^q > r_i^j$ . Thus, for all  $\tau_p^q \prec \tau_i^j$ ,  $r_i^j$  must be modified to satisfy  $r_p^q + C_p^q \leq r_i^j$ , if  $\tau_p^q \prec \tau_i^j$  and  $r_p^q + C_p^q > r_i^j$ . Since  $r_i^j = r_i + O_i^j$ , the offset  $O_i^j$  is increased to increase  $r_i^j$ . Relative deadline  $d_i^j$  is relative to

---

**Algorithm IV-B3.1** Task Release Time Modification
 

---

```

1: repeat
2:   NoChanges ← true
3:
4:   for each  $\tau_p^q \prec \tau_i^j$  do
5:     if  $r_p^q + C_p^q > r_i^j$  then
6:       NoChanges ← false
7:
8:       diff ←  $r_p^q + C_p^q - r_i^j$ 
9:        $O_i^j \leftarrow O_i^j + \text{diff}$ 
10:       $d_i^j \leftarrow d_i^j - \text{diff}$ 
11:       $r_i^j \leftarrow r_i^j + O_i^j$ 
12:
13:      if  $d_i^j < C_i^j$  then
14:        STOP (Deadline Missed)
15:      end if
16:    end if
17:  end for
18: until NoChanges
  
```

---

$O_i^j$ , thus  $d_i^j$  must be decreased by the amount  $O_i^j$  is increased. ■

**(ALG2) Transaction Merge**

Up until now, the transformation algorithm produces separate transactions even if they contain tasks that have precedence dependencies with other tasks from other transactions. This does not respect the modeling of precedence dependencies in [20]. Indeed two tasks with a precedence dependency should be in the same transaction and they should be delayed by a same event that releases the transaction. Two transactions are thus "merged" into one single transaction if there exists a task in one that has a precedence dependency with a task in the other:

$$\exists \tau_p^q, \tau_i^j \mid (\Gamma_i \neq \Gamma_p) \wedge (\tau_p^q \prec \tau_i^j \vee \tau_i^j \prec \tau_p^q)$$

Algorithm IV-B3.2 will merge transactions two by two until there is no more transaction to merge.

---

**Algorithm IV-B3.2** Transaction Merge
 

---

```

1: for each  $\tau_p^q \prec \tau_i^j$  do
2:   if  $\Gamma_p \neq \Gamma_i$  then
3:     for each task  $\tau_i^j$  in  $\Gamma_i$  do
4:       Assign  $\tau_i^j$  to  $\Gamma_p$ 
5:     end for
6:   end if
7: end for
  
```

---

*Proof of Algorithm IV-B3.2:* We remind that tasks of a transaction are related by precedence dependencies and a task in a transaction is released after/by the periodic event that releases the transaction. Let us consider two tasks  $\tau_i^j$  and  $\tau_p^q$ , with  $\tau_p^q \prec \tau_i^j$ . Task  $\tau_i^j$  (resp.  $\tau_p^q$ ) is originally a frame  $F_i^j$  (resp.  $F_p^q$ ). We have  $F_p^q \in [F_p^q]_i^j \Rightarrow T_{G_i} = T_{G_p}$ .  $G_i$  (resp.  $G_p$ ) is transformed into  $\Gamma_i$  (resp.  $\Gamma_p$ ) with period  $T_i$  (resp.  $T_p$ ). We then have  $T_i = T_{G_i} = T_{G_p} = T_p$ . Thus  $\tau_i^j$  and  $\tau_p^q$  are released after/by periodic events of period  $T_i = T_p$ . Since  $\tau_p^q \prec \tau_i^j$ ,  $\tau_i^j$  is released after  $\tau_p^q$ . Thus  $\tau_i^j$  is released after the periodic event after/by which  $\tau_p^q$  is released. Therefore  $\tau_i^j$  and  $\tau_p^q$  are released after/by the same periodic event, that releases transaction  $\Gamma_p$ . Both tasks then belong to  $\Gamma_p$ . ■

**(ALG3) Transaction Release Time Modification**

After merging two transactions into  $\Gamma_m$ , the offset  $O_m^j$  of a task  $\tau_m^j$  (originally denoted  $\tau_o^j$  and belonging to  $\Gamma_o$ ) is still relative to the release time  $r_o$  of  $\Gamma_o$ , no matter the precedence dependencies. In  $\Gamma_m$ , each offset must thus be set relatively to  $r_m$ , the release time of  $\Gamma_m$ . Release time  $r_m$  is computed beforehand. This is done in Algorithm IV-B3.3.

Algorithm IV-B3.3 starts by finding the earliest (minimum) task release time in a merged transaction  $\Gamma_m$  (we remind that  $O_m^j$  is still relative to  $r_o$  at this moment). The earliest task release time becomes  $r_m$ . The offset  $O_m^j$  of each task  $\tau_m^j$  is then be modified to be relative to  $r_m$ .

Note that when transactions are merged and all of them have at least one task released at  $t = 0$ , then Algorithm IV-B3.3 produces the same merged transaction. We will see that this is the case for the transaction resulting from the transformation of our DGMF task set example (Section III-B), which was used to model tasks constrained by a TDMA frame.

---

**Algorithm IV-B3.3** Transaction Release Time Modification
 

---

```

1: for each merged transaction  $\Gamma_m$  do
2:    $r_m \leftarrow +\infty$ 
3:   for each  $\tau_m^j$  in  $\Gamma_m$ , originally in  $\Gamma_o$  do
4:      $r_m \leftarrow \min(r_m, r_o + O_m^j)$ 
5:   end for
6:   for each  $\tau_m^j$  in  $\Gamma_m$ , originally in  $\Gamma_o$  do
7:      $O_m^j \leftarrow r_o + O_m^j - r_m$ 
8:   end for
9: end for
  
```

---

*Proof of Algorithm IV-B3.3:* Let  $\Gamma_m$  be a merged transaction. Tasks in  $\Gamma_m$  were originally in  $\Gamma_o$ . The event that releases  $\Gamma_m$  occurs at  $r_m$ , which must be the earliest release time  $r_m^j$  of a task  $\tau_m^j$  in  $\Gamma_m$ , otherwise the definition of a transaction is contradicted. A task  $\tau_m^j$  should be released at  $r_m^j = r_o + O_m^j$ . Once  $r_m$  is computed, when task offsets have not been modified yet, it is possible to have  $r_o + O_m^j \neq r_m + O_m^j$ . If we assign  $r_m^j \leftarrow r_m + O_m^j$  then  $\tau_m^j$  may not be released at  $r_o + O_m^j$ . This contradicts the fact that  $\tau_m^j$  should be released at  $r_m^j = r_o + O_m^j$ . Therefore  $O_m^j$  must be shortened to be relative to  $r_m$ :  $O_m^j \leftarrow r_o + O_m^j - r_m$ . Since  $r_m = \min_{\tau_m^j \in \Gamma_m} (r_o + O_m^j)$ , the minimum value of  $r_o + O_m^j - r_m$  is 0 and thus the assignment  $O_m^j \leftarrow r_o + O_m^j - r_m$  will never assign a negative value to  $O_m^j$ . ■

c) *Reduce Precedence Dependencies:* In **Step 3.c**, we notice that the transaction set can be simplified by reducing the number of precedence dependency constraints. This could not be done in **Step 3.a**, before **Step 3.b**, because we did not yet know the latest completion time among those of predecessors of a task. Now that offsets have been modified, we can reduce some precedence dependencies. Reducing precedence dependencies has the effect of reducing the number of predecessors/successors of a task.

Algorithm IV-B3.4 loops through tasks with more than 1 predecessor. For a specific task  $\tau_i^j$ , the algorithm reduces predecessors  $\tau_i^q$  that have a global deadline (i.e.  $O_i^q + d_i^q$ ) smaller than the offset  $O_i^j$  of  $\tau_i^j$ .



### Algorithm IV-B3.4 Reduce Precedence Dependencies

```

1: for each task  $\tau_i^j$  with multiple predecessors do
2:   for each  $\tau_i^q \prec \tau_i^j$  do
3:     if  $O_i^q + d_i^q < O_i^j$  then
4:       Remove  $\tau_i^q \prec \tau_i^j$ 
5:     end if
6:   if  $\tau_i^j$  has only one predecessor then
7:     break
8:   end if
9: end for
10: end for

```

*Proof of Algorithm IV-B3.4:* Let us assume  $\tau_i^q \prec \tau_i^j$  and  $O_i^q + d_i^q < O_i^j$ . By definition  $t_0 + O_i^j$  is the earliest release time of a job of  $\tau_i^j$ , corresponding to the job of  $\Gamma_i$  released at  $t_0$ . For the job of  $\Gamma_i$  released at  $t_0$ , the absolute deadline of a corresponding job of  $\tau_i^q$  is  $t_0 + O_i^q + d_i^q$ . The job of  $\tau_i^q$  must finish before  $t_0 + O_i^q + d_i^q$  and  $\tau_i^j$  is released at earliest after  $t_0 + O_i^q + d_i^q$  since  $O_i^q + d_i^q < O_i^j$ . Thus the precedence dependency constraint  $\tau_i^q \prec \tau_i^j$  is already encoded in the relative deadline  $d_i^q$  of  $\tau_i^q$ . Tasks in a transaction are related by precedence dependencies so  $\tau_i^j$  must have at least one predecessor. ■

### C. Transformation Example

The DGMF task set in section III-B is transformed into a transaction  $\Gamma_1$  of period  $T_1 = 20$ . Tasks of transaction  $\Gamma_1$  are defined with parameters shown in Table II (PCP [24] is assumed for computation of  $B_i^j$ ). Tasks are all allocated on CPU1 except  $\tau_1^2$ , which is allocated on CPU2. Fig. 4 shows the precedence dependency graph of tasks. An example of a schedule over 20 time units is shown in Fig. 5.

TABLE II  
TRANSACTION FROM DGMF TRANSFORMATION

	$C_i^j$	$O_i^j$	$d_i^j$	$J_i^j$	$B_i^j$	$prio(\tau_i^j)$
$\tau_1^1$	1	1	3	0	0	1
$\tau_1^2$	1	2	2	0	0	1
$\tau_1^3$	1	3	1	0	0	1
$\tau_1^4$	1	9	3	0	0	1
$\tau_1^5$	4	13	7	0	0	1
$\tau_2^1$	1	0	4	0	0	2
$\tau_2^2$	1	8	4	0	0	2
$\tau_2^3$	1	12	4	0	0	2
$\tau_2^4$	2	16	4	0	3	2
$\tau_3^1$	1	5	1	0	0	1
$\tau_3^2$	1	7	1	0	0	1
$\tau_4^1$	1	4	2	0	0	2
$\tau_4^2$	1	6	2	0	0	2
Tick	0	0	$+\infty$	0	0	0
Critical Sections						
$(\tau_1^3 \text{ R}, 1, 3), (\tau_2^4 \text{ R}, 0, 1)$						

From the task parameters in Table II, the precedence dependency graph in Fig. 4, and the schedule in Fig. 5 we see that a transaction resulting from the transformation of DGMF tasks (modeling tasks constrained by a TDMA frame) has the following characteristics:

- Tree-shaped [22]: A tree-shaped transaction is one where each task may have zero or several successor tasks. Each task may have at most one predecessor. There is an unique

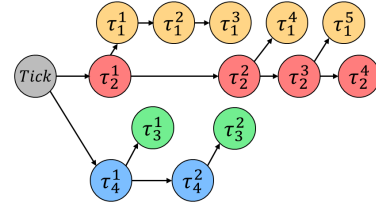


Fig. 4. Tasks Precedence Dependency Graph

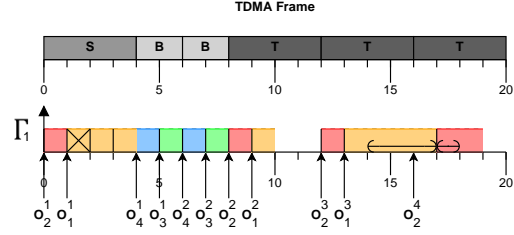


Fig. 5. Transaction Resulted from DGMF Tasks Transformation: Up arrows below timeline are offsets; curved arrows are shared resource allocations; tasks execute on same processor except the crossed task

“root task” in the transaction that does not have any predecessor.

- Tasks may be non-immediate, i.e. a task does not necessarily release immediately its successors.

From these characteristics, we will now propose a suitable scheduling analysis technique for this kind of task set modeling our TDMA system.

### D. Assessing Schedulability of Resulting Transactions

To enforce schedulability of the resulting transactions, we use the schedulability test in [14], based on [22]. The schedulability test is applicable to tree-shaped transactions with *non-immediate tasks*. A *non-immediate task* is one that is not necessarily immediately released by its predecessor. We obtain tree-shaped transactions with *non-immediate tasks* from our DGMF to transaction transformation algorithm, if the DGMF task set has the *Unique Predecessor* property (we remind that DGMF task sets in this article are assumed to have the property):

*Theorem 1:* A DGMF task set with the *Unique Predecessor* property (Property 1) is transformed into a transaction set without tasks that have more than one predecessor.

*Proof:* Let a DGMF task set have the *Unique Predecessor* property. A frame  $F_i^j$  with inter and intra dependencies is transformed into a task  $\tau_i^j$  with multiple predecessor tasks. Task  $\tau_i^j$  has several predecessor tasks that correspond to frames that precede  $F_i^j$ . At most one frame  $F_x^y$  that precedes  $F_i^j$  can have a global deadline (i.e.  $r_x^y + D_x^y$ ) greater than the release time  $r_i^j$  of  $F_i^j$ . By construction, at most one task  $\tau_i^y$  (resulting from  $F_x^y$  and assigned to the same transaction as  $\tau_i^j$ ) that precedes  $\tau_i^j$  can have a global deadline greater than the offset of  $\tau_i^j$  (i.e. only one  $\tau_i^y$  can have  $O_i^y + d_i^y \geq O_i^j$ ). Algorithm IV-B3.4 removes a precedence dependency  $\tau_i^y \prec \tau_i^j$  if  $O_i^j > O_i^y + d_i^y$ . Since there is at most one predecessor  $\tau_i^y$

of  $\tau_i^j$ , such that  $O_i^y + d_i^y \geq O_i^j$ , all other predecessors will be reduced until task  $\tau_i^j$  has at most one predecessor. ■

Note that the release time  $r_i$  of a transaction  $\Gamma_i$  is not used by schedulability tests such as the one in [14]. This has no impact on the analysis since offsets of tasks in  $\Gamma_i$  are relative to  $r_i$ .

## V. EXPERIMENTS

Our proposition is implemented in Cheddar [5], a GPL-licensed open-source real-time scheduling analysis tool. Experiments are conducted on DGMF tasks. These experiments verify the correctness of the transformation, study the performance of the transformation, and evaluate the proposed DGMF analysis technique, compared to existing scheduling analysis techniques, when applied to a real TDMA SRP from Thales.

### A. Transformation Evaluation

To verify the transformation correctness and its time performance, simulation is conducted.

1) *Transformation Correctness*: By using an architecture generator in Cheddar, DGMF task sets are randomly generated. The varying generator parameters are: 2 to 5 DGMF tasks, as many frames as tasks and up to 10 for each number of tasks, 1 to 3 shared resources, as many critical sections as frames, as many precedence dependencies as frames, a DGMF-Period between 10 to 50, and 50% of DGMF tasks with the same DGMF-Period.

From the combination of these varying parameters, 25600 DGMF architecture models are generated. Each of them is transformed to an architecture model with transactions. Both DGMF and transaction models are then simulated over the schedulability interval in [4] and schedules are compared.

For each architecture, we observed that the schedule of the DGMF model is strictly the same as the schedule of the resulting transaction model. This verifies the proofs of the transformation.

2) *Transformation Time Performance*: In the Cheddar implementation, the time complexity of the transformation algorithm depends on two parameters:  $n_F$  the number of frames, and  $n_D$  the number of task dependencies (both precedence and shared resource). The complexity of the transformation is  $O(n_D^2 + n_F)$ . When  $n_F$  is the varying parameter, the complexity of the algorithm should be  $O(n_F)$ . When  $n_D$  is the varying parameter, the complexity of the algorithm should be  $O(n_D^2)$  due to Algorithm IV-B3.1, which has the same complexity as the algorithm in [3].

The experiment in this section checks that the duration of the transformation, implemented in Cheddar, is consistent with these time complexities. Measurements presented below are taken on a Intel Core i5 @ 2.40GHz processor.

Fig. 6 shows the transformation duration by the number of precedence dependencies. The number of frames is set to 1000, the number of DGMF tasks to 100, and the number of shared resource dependencies to 0. Note that it does not matter which dependency parameter (precedence or shared resource) varies to verify the influence of  $n_D$ , since all dependencies

are iterated through once in the Cheddar implementation and precedence dependency has more impact on the transformation duration. Since there are 1000 frames and 100 DGMF tasks, the minimum number of precedence dependencies starts at 900, due to intra dependencies. From Fig. 6 we see that the transformation duration is polynomial when the number of precedence dependencies vary. This result is consistent with the complexity, which is  $O(n_D^2)$  when  $n_D$  is the varying parameter.

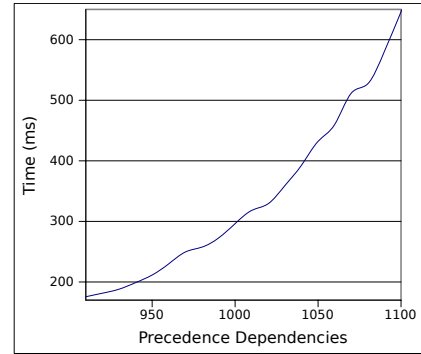


Fig. 6. Transformation Response Time by Number of Precedence Dependencies

Fig. 7 shows the transformation response time by the number of frames. The number of precedence dependencies is set to 0 (i.e. no intra dependencies either) and the other parameters remains the same. From Fig. 7 we see that the response time is polynomial when the number of frames varies. One can think that this result is inconsistent with the time complexity of the algorithm, which is  $O(n_F)$  when  $n_F$  is the varying parameter. In practice, the implementation in Cheddar introduces a loop to verify that a task is not already present in the system's task set. Thus the time complexity of the implementation is  $O(n_F^2)$ .

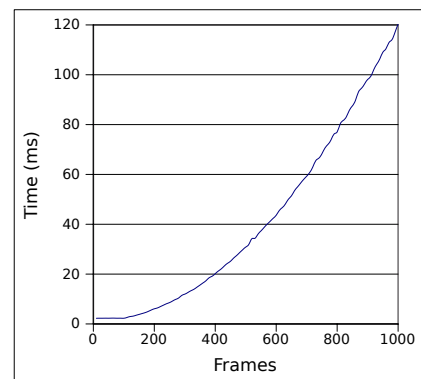


Fig. 7. Transformation Response Time by Number of Frames

Overall we see that a system with no dependency, 1000 frames, and 100 DGMF tasks, takes about 120ms to be transformed on the PC used for the experiment. A system with 1100 precedence dependencies, 1000 frames, and 100 DGMF tasks, takes less than 650ms to be transformed. The



transformation duration is acceptable for our needs. Indeed for a typical TDMA frame handled by Thales, with 13 slots (1S, 4B, 8T) and 10 critical tasks, there would be a maximum of 130 frames ( $13 \times 10$ ), and 237 precedence dependencies ( $10 \times 13 - 10 + 9 \times 13$ ) if all tasks are part of a same end-to-end flow released at each slot.

### B. Experiment on a TDMA SRP from Thales

We now apply the DGMF task model to the modeling and scheduling analysis of a real TDMA SRP. The results given by DGMF analysis are compared to results given by GMF Worst Case Response Time (WCRT) analysis [25] and periodic task WCRT analysis [10]. A Cheddar model of the full case-study (8 DGMF tasks, 44 frames) can be downloaded at the Cheddar website<sup>1</sup>. For the sake of space, let us consider a simple TDMA frame with two slots shown in Fig. 8.

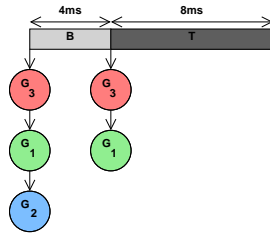


Fig. 8. Experiment TDMA SRP

In our system there are three tasks:  $G_1$ ,  $G_2$  and  $G_3$ . Task  $G_3$  is released at the beginning of each slot. After it finishes execution, it releases  $G_1$ .  $G_1$  releases  $G_2$  at the first  $B$  slot but not at the  $T$  slot.  $G_3$  and  $G_1$ , when released at a slot, must finish before the end time of the slot.  $G_2$  when released at the  $B$  slot, must finish before the end time of slot  $T$ . Tasks are scheduled by a preemptive fixed priority scheduler and run on a single processor. PCP is used for shared resource synchronization. Tasks have parameters shown in Table III. Task priorities are in highest priority first order (e.g. a priority level 3 task has a higher priority than a priority level 1 task). Execution times come from a real SRP application. Time units are in  $\mu s$ .

To compare the DGMF analysis with the GMF WCRT analysis and the periodic task WCRT analysis, we also model these tasks in the GMF and periodic models. Since shared resource synchronization cannot be modeled in the GMF model, we do not consider them. If the GMF WCRT analysis is still more pessimistic without shared resources, then pessimism will not be improved with shared resources. As for the periodic model, the tasks were considered as sporadic, and then modeled as periodic. Their longest frame execution time is taken as their WCET. Their smallest min-separation time between two frames is taken as their period.

Computed WCRTs are shown in Table IV. Headers represent the task model. A response time with "/" means a deadline was missed.

<sup>1</sup>[http://beru.univ-brest.fr/svn/CHEDDAR/trunk/project\\_examples/wcdops+\\_nimp/tdma\\_mac/](http://beru.univ-brest.fr/svn/CHEDDAR/trunk/project_examples/wcdops+_nimp/tdma_mac/)

TABLE III  
EXPERIMENT TASK SET

(D)GMF		$E_i^j$	$D_i^j$	$P_i^j$	$[F_P^q]_i^j$
$G_1, prio(G_1) = 1$	$F_1^1$ $F_1^2$	955 1874	4000 8000	4000 8000	$F_3^1$ $F_3^2$
$G_2, prio(G_2) = 2$	$F_2^1$	5722	12000	12000	$F_1^1$
$G_3, prio(G_3) = 3$	$F_3^1$ $F_3^2$	986 986	4000 8000	4000 8000	
Periodic Model					
		$C_i$	$d_i$	$T_i$	$p_i$
$G_1$		1874	4000	4000	1
$G_2$		5722	12000	12000	2
$G_3$		986	4000	4000	3

TABLE IV  
EXPERIMENT RESPONSE TIMES

		DGMF WCRT	GMF WCRT	Periodic WCRT
$G_1$	$F_1^1$ $F_1^2$	1941 6523	/	/
$G_2$	$F_2^1$	8649	7694	7694
$G_3$	$F_3^1$ $F_3^2$	986 986	986 986	986

From the WCRTs we see that DGMF analysis determines that no deadlines are missed. This is not the case for the other two analysis techniques.

For  $G_2$ , GMF WCRT analysis gives a lower WCRT than DGMF analysis but this value is underestimated. Indeed GMF WCRT analysis considers that  $F_2^1$  is only interfered by  $F_3^1$  and  $F_3^2$ , without considering the fact that  $F_2^1$  is released after  $F_1^1$ . In conclusion DGMF analysis determines a schedulable system, and precedence dependencies must be considered by the analysis, to not underestimate WCRTs.

## VI. RELATED WORK

Several works are related either to scheduling analysis of TDMA communication systems or to the DGMF task model proposed in this article.

TDMA systems are networked systems for which there exist formal methods, like network calculus [13], to bound end-to-end response times of messages in the network. TDMA systems are also a special case of time-triggered systems [12] for which there exists feasibility tests [18]. Both approaches do not consider shared resources and the event-triggered [12] aspect of SRPs, through precedence dependencies.

Transactions used in this article were initially proposed by [27] to compute message end-to-end response times in a distributed system where tasks communicate through a TDMA bus. [27] thus models the holistic behavior of the TDMA bus to assess schedulability. The author's approach is based on the classic periodic task model, in which task parameter values do not vary and tasks do not have real precedence dependencies (i.e. their offset and jitter are static). In our approach, we need to model depending tasks that have variable parameters (e.g. execution time) due to functional properties related to a TDMA frame.

Schedulability tests that make use of precedence dependency conflicts [20], or execution-time dependencies [17], have been proposed for linear transactions (tasks can have at most one successor and predecessor). These tests cannot be applied to transactions resulting from DGMF transformation, since they are tree-shaped.

In [8] the authors propose a model for multi-event synchronization. Their approach is to transform a system with tasks having multiple predecessors and successors, into a set of linear transactions and then use tests for linear transactions. This approach cannot be applied to transactions resulting from DGMF transformation, since these transactions are tree-shaped and they have non-immediate tasks.

Tests for tree-shaped [22], [9] and graph-shaped [11] transactions have been proposed. These tests cannot be applied to transactions resulting from DGMF transformation since these transactions have non-immediate tasks.

In [26] the authors propose non-cyclic GMF tasks to model behavior of tasks in software radio modems. Their work does not consider TDMA constrained tasks and task dependencies. In [6] the authors propose an optimal resource sharing protocol for GMF tasks. In our work, we also focus on precedence dependencies that is not handled by [6].

In [7] Semaphore Precedence Constraint (SPC) is proposed. A SPC models a multi-rate communication that follows a regular repetitive pattern (e.g. a task depends on every third instance of another task). This pattern is not compliant with our task release requirements. Furthermore the authors use linear transactions (which cannot be applied to the systems we investigate) and they do not focus on variable task parameters.

## VII. CONCLUSION

In this article we expected to improve scheduling analysis of systems with TDMA communications and task dependencies. A TDMA Software Radio Protocol was used as an illustration. The DGMF task model was proposed to model a TDMA system. To assess schedulability of DGMF tasks, we proposed to transform DGMF tasks to tree-shaped transactions with non-immediate tasks for which we previously proposed a schedulability test. Experiments were done on a TDMA SRP from Thales and showed that the DGMF model gives less pessimistic schedulability results than both the GMF model and the periodic model.

GMF, on which DGMF is based, was initially proposed for the multimedia domain. In the future, we will investigate the applicability of DGMF to such domains.

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