Multi-Objective Optimization and Obstacle Avoidance Strategies for Remotely Operated Vehicles Path Planning

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August 2024

Abstract:
This paper explores the optimization of path planning for Remotely Operated Vehicles through the development of a multi-objective optimization model and the implementation of obstacle avoidance strategies.
Key words:
Route Optimization, Weighted Sum Method, Dijkstra Algorithm, Dynamic Trajectory Adjustment

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Chapter 1

Introduction

Remotely Operated Vehicles (ROVs) have revolutionized underwater exploration and operations, playing a pivotal role in various industries, including marine research, oil and gas, and underwater construction. These machines are equipped with advanced sensors and technologies, enabling them to navigate complex underwater environments, perform inspections, and carry out tasks that would be hazardous or impossible for human divers. The impact of ROVs extends beyond mere operational efficiency; they contribute significantly to environmental monitoring, resource management, and the advancement of scientific knowledge about marine ecosystems.

As ROVs are deployed in increasingly challenging environments, the need for effective navigation and obstacle avoidance mechanisms becomes paramount. These vehicles often operate in dynamic settings where unexpected obstacles, such as underwater waste or marine life, can pose significant risks to their mission objectives. Therefore, developing robust algorithms and optimization models that allow ROVs to adapt their trajectories in real-time is essential for ensuring safe and efficient operations. The integration of obstacle avoidance strategies not only enhances the operational capabilities of ROVs but also minimizes the risk of damage to both the vehicle and the surrounding environment.

Optimization of path planning plays a crucial role in enhancing the performance of ROVs. At its core, optimization involves finding the best solution to a problem within a defined set of constraints and objectives. In the case of ROVs, this means determining the most efficient path that the vehicle can take while avoiding obstacles and full-filling operational constraints such as energy consumption and mission timelines. The optimization process can be complex, as it must account for multiple objectives, including minimizing travel distance while prioritizing completion of different tasks and minimizing risk caused by environmental changes.

Multi-objective optimization (MOO) techniques are particularly relevant in this context, as they allow for the simultaneous consideration of conflicting objectives. For instance, an ROV may need to balance the trade-off between minimizing travel time and maximizing the quality of data collected during its mission or minimizing travel distance while maximizing number of tasks that need to be completed. The weighted sum method is one approach that can be employed to tackle such multi-objective problems. By assigning weights to each objective, decision-makers can prioritize their goals and explore various trade-offs, ultimately leading to more informed and effective operational strategies.

In the process of finding path for avoiding the obstacle, the application of optimization techniques becomes even more critical. ROVs must be equipped with algorithms that enable them to detect obstacles in real-time and adjust their paths accordingly. This requires a dynamic optimization model that can process incoming data from sensors and imaging systems, allowing the ROV to make quick decisions about its trajectory. The Dijkstra Algorithm is one of the well-known path-finding algorithm that can be adapted for use in ROVs to identify the shortest and safest route while avoiding detected obstacles.

Chapter 2

Introduction to Optimization

2.1 Definition and Basic Contepts

Optimization is a fundamental concept that pervades various fields, from engineering and economics to logistics and machine learning. The essence of optimization is to find the best possible solution to a problem given a set of constraints and objectives. Formally, an optimization problem involves determining the optimal value of a function within a given domain. This section provides a general overview of optimization, followed by a more detailed discussion of linear programming, a specific and widely used optimization technique.

Optimization problems can be broadly categorized into several types based on the nature of the objective function, the constraints, and the decision variables. The major categories include linear, nonlinear, integer, combinatorial, and dynamic optimization.

We begin by defining the most generalized form of the optimization problem.

Definition 2.1.1. An optimization problem can be expressed as:

$$\min_{x \in X} f(x)$$

where $f(x)$ is the objective function to be minimized, and X is the feasible region defined by a set of constraints. The feasible region X is the set of all x that satisfy the constraints of the problem.

An optimization problem typically involves either minimizing or maximizing an objective function. This means that sometimes our goal is to find not just the lowest value but also the highest value of the objective function, which requires maximizing it.

To address both types of problems in a consistent way, we can use a simple mathematical principle:

$$\max f(x) \Leftrightarrow \min(-f(x))$$

This principle shows that any problem where we need to maximize a function can be turned into a minimization problem by taking the negative of the function, and vice versa. Therefore, most optimization problems are commonly formulated as minimization problems for simplicity and convenience.

2.2 Linear Programming

Linear programming (LP) is a powerful mathematical method for determining the best outcome in a mathematical model whose requirements are represented by linear relationships. It has numerous applications in various industries, including manufacturing, transportation, finance, telecommunications, and military planning.

Definition 2.2.1. A linear programming problem is formulated as:

$$\min_{x \in \mathbb{R}^n} \mathbf{c}^T \mathbf{x}$$

subject to:

$$\mathbf{A}\mathbf{x} \leq \mathbf{b}$$

$$\mathbf{x} \geq 0$$

where $\mathbf{c}$ is an n-dimensional vector of coefficients for the objective function, $\mathbf{A}$ is an $m \times n$ matrix of coefficients for the constraints, $\mathbf{b}$ is an m-dimensional vector of constraint bounds, and $\mathbf{x}$ is the vector of decision variables.

As we can see from definition above, in linear programming, both the objective function and the constraints are linear. This means that the objective function, which is to be either maximized or minimized, is a linear combination of decision variables. Similarly, the constraints, which define the feasible region, are linear equations or inequalities involving the decision variables. The feasible region, defined by the intersection of linear constraints, forms a convex polytop. This convexity implies that any local optimum is also a global optimum, which is a significant advantage in finding the best solution efficiently. Linear programming benefits from a well-established theoretical foundation and efficient algorithms, such as the Simplex method and interior-point methods, which make solving these problems relatively straightforward [1].

Example 2.2.1 (Production Optimization)**.** A manufacturer produces two different products X_1 and X_2 using three machines M_1, M_2, and M_3. Each machine can be used only for a limited amount of time. The production times of each product on each machine vary: product X_1 requires 1 hour on machine M_1, 1 hour on machine M_2, and 3 hours on machine M_3; product X_2 requires 1 hour on machine M_1, 4 hours on machine M_2, and 1 hour on machine M_3. Machine M_1 has a maximum working time of 10 hours, machine M_2 is restricted to 20 hours, and machine M_3 is limited to 12 hours to ensure optimal operation and avoid excessive wear. The objective is to maximize the combined time of utilization of all three machines.

Every production decision must satisfy the constraints on the available time. In particular, we have:

$$\begin{aligned} x_1 + x_2 &\leq 10 \\ x_1 + 4x_2 &\leq 20 \\ 3x_1 + x_2 &\leq 12 \end{aligned}$$

where x_1 and x_2 denote the production levels. The combined production time of all three machines is:

$$f(x_1, x_2) = 3x_1 + 6x_2.$$

Thus, the problem in compact notation has the form:

$$\begin{array}{ll} \text{maximize} & \mathbf{c}^T\mathbf{x} \\ \text{subject to} & \mathbf{A}\mathbf{x} \leq \mathbf{b} \\ & \mathbf{x} \geq 0, \end{array}$$

where

$$\mathbf{c}^T = [3, 6],$$
$$\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix},$$
$$\mathbf{A} = \begin{bmatrix} 1 & 1 \\ 1 & 4 \\ 3 & 1 \end{bmatrix},$$
$$\mathbf{b} = \begin{bmatrix} 10 \\ 20 \\ 12 \end{bmatrix}.$$

Definition 2.2.2. Any vector $\mathbf{x}$ that yields the minimum value of the objective function $\mathbf{c}^T\mathbf{x}$ over the set of vectors satisfying the constraints $\mathbf{Ax} = \mathbf{b}, \mathbf{x} \geq 0$, is said to be an **optimal feasible solution**.

Definition 2.2.3. An m-element subset B of $\{1, \ldots, n\}$ is said to be a **basis** (with respect to matrix A) if the columns of A indexed by the elements in B are linearly independent.

We say that $\mathbf{x}^* \in \mathbb{R}^n$ is a **basic solution** to the system $A\mathbf{x} = \mathbf{b}$ if there exists a basis B such that

(i) $A\mathbf{x}^* = \mathbf{b}$;

(ii) $x_j^* = 0$ for all $j \notin B$.

Definition 2.2.4. An optimal feasible solution that is basic is said to be an **optimal basic feasible solution**.

Theorem 2.2.1 (Fundamental Theorem of LP)**.** Consider a linear program in standard form.

(i) If there exists a feasible solution, then there exists a basic feasible solution;

(ii) If there exists an optimal feasible solution, then there exists an optimal basic feasible solution.

2.3 Multi-objective Optimization

In recent years, the field of multi-objective optimization has gained significant attention across various domains, including engineering, logistics, finance, and environmental management. Multi-objective optimization involves the simultaneous optimization of two or more objectives, which is a common scenario in real-world applications.

2.3.1 Multi-objective Optimization Problem

As in a single-objective optimization problem, the multi-objective optimization problem may contain a number of constraints which any feasible solution (including all optimal solutions) must satisfy. Thus, any multi-objective optimization problem can be represented by the following general mathematical model:

$$\begin{array}{lrl}
\text{min} & \mathbf{f}(\mathbf{x}) = [f_1(\mathbf{x}), f_2(\mathbf{x}), \ldots, f_m(\mathbf{x})]^T & \\
\text{subject to} & g_i(\mathbf{x}) \geq 0, & i = 1, 2, \ldots, p \\
 & h_j(\mathbf{x}) = 0, & j = 1, 2, \ldots, q \\
 & x_i^{(\min)} \leq x_i \leq x_i^{(\max)}, & i = 1, 2, \ldots, n \\
 & \mathbf{x} = [x_1, x_2, \ldots, x_n]^T \in \mathcal{Q} &
\end{array}$$

where m is the number of objective functions, $\mathcal{Q}$ is the n-dimensional search space defined by the lower bounds $\mathbf{x}^{(\min)} = [x_1^{(\min)}, x_2^{(\min)}, \ldots, x_n^{(\min)}]^T$ and upper bounds $\mathbf{x}^{(\max)} = [x_1^{(\max)}, x_2^{(\max)}, \ldots, x_n^{(\max)}]^T$ of decision variables $\mathbf{x}$. The constraints $g_i(\mathbf{x}) \geq 0$ and $h_j(\mathbf{x}) = 0$ represent p inequality constraints and q equality constraints, respectively. If $p = q = 0$, the problem simplifies to an unconstrained multi-objective optimization problem.

Example 2.3.1 (Multi-Objective Production Optimization)**.** A manufacturer produces two different products X_1 and X_2 using three machines M_1, M_2, and M_3. Each machine can be used only for a limited amount of time. Production times of each product on each machine are given in Table 1. The objective is to maximize the combined production time of utilization of all three machines as well as to maximize the profit generated by the production of these products.

Machine	X_1 (hours)	X_2 (hours)
M_1	2	3
M_2	4	1
M_3	3	2

Table 2.1: Production times for X_1 and X_2 on machines M_1, M_2, and M_3

The time constraints for each machine are as follows:

$$\begin{aligned} 2x_1 + 3x_2 &\leq 15 \qquad \text{(Machine } M_1 \text{ time constraint)} \\ 4x_1 + x_2 &\leq 10 \qquad \text{(Machine } M_2 \text{ time constraint)} \\ 3x_1 + 2x_2 &\leq 12 \qquad \text{(Machine } M_3 \text{ time constraint)} \end{aligned}$$

The combined production time of all three machines is given by the function:

$$f_1(x_1, x_2) = 2x_1 + 3x_2 + 4x_1 + x_2 + 3x_1 + 2x_2 = 9x_1 + 6x_2$$

The profit generated by producing these products is given by:

$$f_2(x_1, x_2) = 5x_1 + 7x_2$$

Thus, the problem in compact notation has the form:

$$\begin{aligned} \text{maximize} \qquad & [f_1(\mathbf{x}), f_2(\mathbf{x})]^T \\ \text{subject to} \qquad & \mathbf{Ax} \leq \mathbf{b} \\ & x_1, x_2 \geq 0, \end{aligned}$$

where:

$$\begin{aligned} f_1(x_1, x_2) &= 9x_1 + 6x_2, \\ f_2(x_1, x_2) &= 5x_1 + 7x_2 \\ \mathbf{A} &= \begin{bmatrix} 2 & 3 \\ 4 & 1 \\ 3 & 2 \end{bmatrix}, \\ \mathbf{b} &= \begin{bmatrix} 15 \\ 10 \\ 12 \end{bmatrix}, \\ \mathbf{x} &= \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \end{aligned}$$

2.3.2 Multi-objective Optimization Solutions

In multi-objective optimization problems, the challenge lies in defining the solutions. From a mathematical standpoint, there is not a single solution but rather a set of solutions. In 1951, Koopmans introduced the concept of Pareto efficiency, which describes the solution set under partial order rather than total order. A solution is considered Pareto optimal if no other solution can improve one objective without degrading another. This concept is crucial in decision-making processes where multiple criteria must be considered.

Definition 2.3.1 (Feasible Solution)**.** A solution vector $\mathbf{x} \in \mathcal{Q}$ is defined as a feasible solution if it satisfies all the inequality and equality constraints for $i = 1, 2, \ldots, p$ and $j = 1, 2, \ldots, q$. Otherwise, it is an infeasible solution.

All feasible solutions constitute the feasible domain $\mathcal{U}$, and all infeasible solutions constitute the infeasible domain $\mathcal{U}'$. Clearly, $\mathcal{U} \cup \mathcal{U}' = \mathcal{Q}$, where $\mathcal{U} \subseteq \mathcal{Q}$ and $\mathcal{U}' \subseteq \mathcal{Q}$.

In other words, feasible solution is one that meets all the constraints imposed by the problem. The feasible domain is the set of all such solutions, while the infeasible domain is the set of solutions that do not meet the constraints.

In the decision variable space (space of all possible values of decision variables), a solution **a** is said to dominate another solution **b** if **a** is no worse than **b** in all objectives and strictly better in at least one objective.

Definition 2.3.2 (Decision Variable Domination)**.** For two vectors $\mathbf{a} = [a_1, a_2, \ldots, a_n]^T$ and $\mathbf{b} = [b_1, b_2, \ldots, b_n]^T$ in the decision variable space, **a** is said to dominate **b** (denoted as $\mathbf{a} \prec \mathbf{b}$) if:

(i) $\forall i \in \{1, 2, \ldots, m\}\ f_i(\mathbf{a}) \leq f_i(\mathbf{b})$

(ii) $\exists j \in \{1, 2, \ldots, m\}\ f_j(\mathbf{a}) < f_j(\mathbf{b})$

In the objective function space, a point **g** dominates another point **h** if **g** is no worse than **h** in all objectives and strictly better in at least one objective.

Definition 2.3.3 (Objective Function Domination)**.** For two vectors $\mathbf{g} = [g_1, g_2, \ldots, g_m]^T$ and $\mathbf{h} = [h_1, h_2, \ldots, h_m]^T$ in the objective function space, $\mathbf{g}$ is said to dominate $\mathbf{h}$ (denoted as $\mathbf{g} \prec \mathbf{h}$) if:

(i) $\forall i \in \{1, 2, \ldots, m\}$, $g_i \leq h_i$

(ii) $\exists j \in \{1, 2, \ldots, m\}$, $g_j < h_j$

Definition 2.3.4 (Pareto Optimal Solution)**.** A vector $\mathbf{x}^* = [x_1^*, x_2^*, \ldots, x_n^*]^T \in \mathcal{Q}$ is a Pareto optimal solution if:

$$\forall \mathbf{x} \in \mathcal{Q},\ \mathbf{x} \neq \mathbf{x}^* \Rightarrow \mathbf{f}(\mathbf{x}) \not\prec \mathbf{f}(\mathbf{x}^*)$$

The set of all Pareto optimal solutions is called the Pareto optimal set, denoted as $\mathcal{PS}^*$.

In other words, a Pareto optimal solution is one where no other solution in the feasible domain can improve any objective without causing a degradation in at least one other objective. The set of all Pareto optimal solutions is known as the Pareto optimal set, denoted as PS^*.

Definition 2.3.5 (Pareto Optimal Front)**.** The Pareto optimal set represented in the objective function space is called the Pareto optimal front, denoted as:

$$\mathcal{PF}^* = \{\mathbf{f}(\mathbf{x}) \,|\, \mathbf{x} \in \mathcal{PS}^*\}$$

The Pareto optimal front is the set of objective vectors corresponding to the Pareto optimal solutions. It represents the trade-offs between different objectives in the objective function space.

Multi-objective optimization methods aim to find solutions that are as close as possible to the Pareto optimal front and are uniformly distributed. Such methods should exhibit good convergence and diversity. Additionally, the solutions should be numerous to ensure a wide range of options for decision-makers. Once the Pareto optimal set is found, decision-makers can select the final solution based on specific optimization problems or personal preferences. A diverse and extensive set of solutions allows for better comparison and selection according to various criteria and preferences.

2.3.3 Weighted Sum Method

One of the most widely used approaches to tackle multi-objective optimization problems is the weighted sum method.

This method transforms the multi-objective problem into a single-objective problem by assigning weights to each objective function, reflecting their relative importance. The weighted sum of the objectives is then optimized, allowing for the exploration of different trade-offs by varying the weights. This approach is particularly appealing due to its simplicity and ease of implementation.

However, it has its limitations, such as the potential to miss non-convex regions of the Pareto front and the challenge of selecting appropriate weights that accurately represent the decision-maker's preferences.

Mathematically, general formulation of the weighted sum method for a multi-objective optimization problem can be expressed as follows:

$$\begin{array}{lrl}
\text{min} & f(\mathbf{x}) = \sum_{m=1}^{M} w_m f_m(\mathbf{x}) & \\
\text{subject to} & g_j(\mathbf{x}) \geq 0, & j = 1, 2, \ldots, J \\
 & h_k(\mathbf{x}) = 0, & k = 1, 2, \ldots, K \\
 & x_i^{(L)} \leq x_i \leq x_i^{(U)}, & i = 1, 2, \ldots, n
\end{array}$$

where:

(i) $\mathbf{x}$ is the vector of decision variables.

(ii) $f_m(\mathbf{x})$ is the m-th objective function.

(iii) w_m is the weight assigned to the m-th objective function, with $w_m \geq 0$ and $\sum_{m=1}^{M} w_m = 1$.

(iv) $g_j(\mathbf{x})$ are the inequality constraints.

(v) $h_k(\mathbf{x})$ are the equality constraints.

(vi) $x_i^{(L)}$ and $x_i^{(U)}$ are the lower and upper bounds on the decision variables.

The weights w_m are user-supplied and represent the priority or importance of each objective function.

One of the key strengths of this method lies in its simplicity and flexibility. The weighted sum method is straightforward and easy to implement. It converts a multi-objective problem into a single-objective problem, which can be solved using standard optimization techniques. Furthermore, by adjusting the weights, decision-makers can explore different trade-offs between the objectives. This allows for a customized approach depending on the relative importance of each objective.

On the other hand, choosing appropriate weights to obtain a desired Pareto-optimal solution can be challenging. The solution is sensitive to the choice of weights, and inappropriate weights may lead to sub-optimal solutions. Additionally, in cases where the objective space is non-convex, the weighted sum method may fail to find certain Pareto-optimal solutions. This is because the method relies on linear combinations of the objectives, which may not capture the true trade-offs in a non-convex space [2].

2.4 Mathematical Model

In this section we are designing a model to determine the optimal route for a Remotely Operated Vehicle (ROV) in an offline setting, considering various factors that influence the route choice.

The ROV's mission entails visiting a series of stations, each with a specific priority for visitation, while taking into account the distance between stations and the associated environmental risks. The objective is to minimize the total distance traveled, prioritize stations based on their urgency (with 1 being the highest priority and 5 the lowest) and precedence, and manage risks related to certain stations. This must be accomplished within a set of constraints to ensure an efficient and feasible route.

By employing the weighted sum method, we combine these objectives into a single, flexible objective function. The constants α, β, and γ play a crucial role in balancing the different objectives, allowing for a tailored approach depending on the specific mission requirements.

The model's design is not only practical but also adaptable, with the ability to adjust the weights to explore different trade-offs and optimize the ROV's route according to the decision-maker's preferences. This foundation paves the way for further refinement and application in real-world scenarios, where the balance between these competing objectives is critical to the success of ROV missions.

By minimizing the total distance, prioritizing stations with higher urgency, and considering the risks associated with environmental conditions, this model provides a comprehensive and flexible tool for offline ROV route optimization.

2.4.1 Data

To build our model, we first need to define and formalize the data that will be used. Let us assume we have gathered the necessary information and now introduce the notation that will represent the data:

(i) B: set of all stations.

(ii) $S = |B| < \infty$: total number of stations.

(iii) N: minimum number of stations to visit ($0.5\ S$).

(iv) d_{ij}: distance between station i and station j.

(v) p_i: priority of station i, lower values indicate higher priority.

(vi) P: set of precedence ordered pairs (i, j) such that beacon i must be visited before beacon j

(vii) r_i: the risk of extreme weather conditions at station i

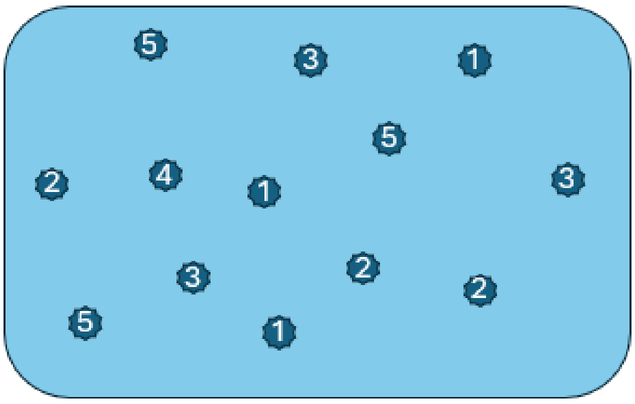


Figure 2.1: Example of data: Search area with station ranked by visitation priority

2.4.2 Decision Variables

The decision variables x_{ij} and u_i are essential components of the model, with each serving a distinct purpose in the optimization process:

(i) x_{ij}: binary variable indicating if the route goes from station i to station j.

(ii) u_i: auxiliary variables for sub-tour elimination.

2.4.3 Objective Function

Using the weighted sum method, the objective function in the model is expressed as:

$$\alpha \sum_{i\in B}\sum_{j\in B} d_{ij}\cdot x_{ij} + \beta \sum_{i\in B}\sum_{j\in B} p_i \cdot x_{ij} + \gamma \sum_{i\in B}\sum_{j\in B} r_i \cdot x_{ij}$$

Here is an explanation of each part of the objective function:

Term

$$\sum_{i\in B}\sum_{j\in B} d_{ij}\cdot x_{ij}$$

focuses on minimizing the total distance in the route. Here, d_{ij} represents the distance between station i and station j, and x_{ij} is a binary variable indicating whether the route from i to j is taken (1 if taken, 0 otherwise).
Term

$$\sum_{i \in B} \sum_{j \in B} p_i \cdot x_{ij}$$

incorporates the priority or urgency of each station into the optimization. Here, p_i represents the priority of station i, with lower values indicating higher priority. The product $p_i \cdot x_{ij}$ thus reflects the priority Finally, term

$$\sum_{i \in B} \sum_{j \in B} r_i \cdot x_{ij}$$

accounts for additional factors represented by r_i, which include various risks associated with visiting station i. The product $r_i \cdot x_{ij}$ incorporates these risks into the route planning, ensuring that the model considers potential challenges or disadvantages when determining the optimal path. Stations situated in environmentally sensitive or hazardous areas might have higher r_i values to account for potential environmental impacts or challenges.

The formulation of objective function effectively combines the three objectives—minimizing distance, prioritizing important stations, and minimizing risk—into a single objective. The constants α, β, and γ can be adjusted to achieve the desired balance between these competing objectives, with the constraint that their sum must equal one. This adjustment allows for an analysis of how their values impact the overall objective function value.

2.4.4 Constraints

Constraints are given by:

$$\sum_{i\in B}\sum_{j\in B} x_{ij} \geq N \tag{2.1}$$

$$\sum_{j\in B} x_{ij} \leq 1 \quad \forall i \in B \tag{2.2}$$

$$\sum_{i\in B} x_{ij} \leq 1 \quad \forall j \in B \tag{2.3}$$

$$u_i - u_j + S \cdot x_{ij} \leq S - 1 \quad \forall i, j \in B,\ i \neq j \tag{2.4}$$

$$u_i \geq 0 \quad \forall i \in B \tag{2.5}$$

$$x_{ij} \in \{0, 1\} \quad \forall i, j \in B \tag{2.6}$$

$$p_i \in \{1, 2, 3, 4, 5\} \quad \forall i \in B \tag{2.7}$$

$$u_i \leq u_j \quad \forall (i, j) \in P \tag{2.8}$$

- (1): Visit at least 50% of the stations
- (2): Each visited station must be exited exactly once
- (3): Each visited station must be entered exactly once
- (4): Sub-tour elimination
- (5): This constraint ensures that the values of are appropriate for eliminating sub-tours
- (6): constraint on the variable's values
- (7): constraint on the variable's values
- (8): ensures that the auxiliary variable for station i is less than or equal to that of station j, effectively enforcing the visit order

Chapter 3

Obstacle Avoidance Using Dijkstra Algorithm

Now, we are addressing the problem of obstacle avoidance for a remotely operated vehicle in a 3D underwater environment. In the context of remotely operated vehicles (ROVs) performing underwater navigation, an effective online obstacle avoidance mechanism is crucial for maintaining safe and efficient travel. This process is particularly significant when the ROV, which follows a pre-planned offline path, encounters unexpected obstacles in its environment. The essence of this problem is to dynamically adjust the ROV's trajectory to navigate around such obstacles while ensuring that the detour is minimal and sensible and that ROV returns to the planned path. This mechanism becomes active when the ROV detects an obstacle, which is identified through image processing techniques.

3.1 Problem Definition

The ROV is tasked with following a predetermined path that optimally visits its various stations or waypoints. However, the path planning conducted offline does not account for unforeseen obstacles that might suddenly appear in the ROV's path during its operation. To address this, the ROV must employ an obstacle avoidance mechanism that can adapt to real-time changes in the environment.

The ROV is equipped with image-capturing technology that provides continuous visual data of its surroundings. At discrete intervals, denoted as Δt, the ROV updates its memory with new image data. The image processing system analyzes these images to detect the presence of obstacles. If an obstacle is detected, it is represented as a polygon with n sides, which approximates the shape and extent of the obstacle. This polygonal representation allows for a more precise definition of the obstacle's location and dimensions.

The operational procedure involves the following steps:

1. The ROV relies exclusively on images captured from its onboard sensors to perceive its surroundings. At discrete time intervals, denoted as Δt, the ROV updates its environmental memory with new data acquired through these images.

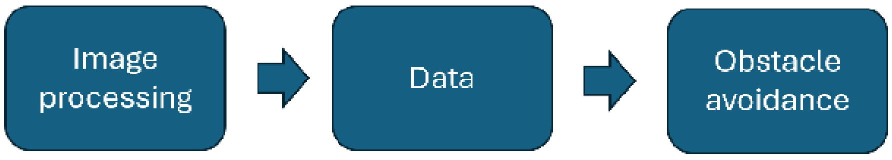


Figure 3.1: Data is gathered from image processing and used for obstacle avoidance strategies

2. When an image procesing indicates the presence of an obstacle, the ROV initiates an obstacle avoidance protocol. This involves analyzing the captured image to identify and locate the obstacle. The result of this analysis is a polygon with n sides that approximates the obstacle's shape and position in the environment. Vertices of the n - sided polygon are 3-dimensional points that are input data of graph search algorithm for obstacle avoidance. Meaning, at each time step Δt, a graph $G(V, E)$ is updated where:

   (i) V represents the set of vertices (3D coordinates) of the polygons.

   (ii) E represents the edges, which are the Euclidean distances between these vertices.

   Since the positions of the vertices are updated at each time step, the graph is dynamic and needs to be reconstructed continually.

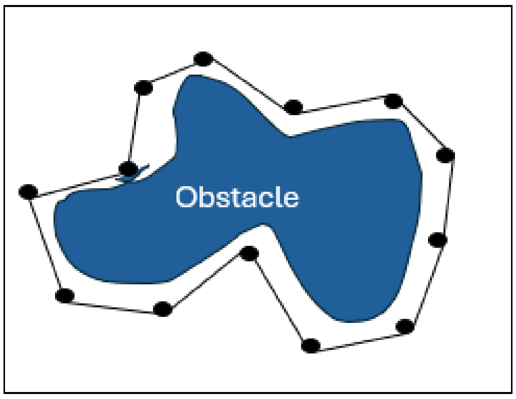


Figure 3.2: Polygon with sides that approximates the obstacle's shape

To ensure accurate obstacle avoidance, we need to address the fact that not every vertex of the polygon will be used as a node in the graph due to limitations in the captured image. Specifically, the image may not encompass the entirety of the obstacle, leading to incomplete data. Vertices of the polygon located along the periphery of the captured image are particularly problematic. These vertices suggest that the captured image does not cover the entire extent of the obstacle, indicating that the obstacle likely extends beyond the edges of the captured picture.

Therefore, these vertices that are on or closed to the periphery of the captured image are disregarded in the graph construction process. This is because including them could lead to an inaccurate representation of the obstacle's shape and position. When vertices on the edge of the image are included, there is a risk that the polygon's boundaries are not correctly aligned with the actual obstacle. Since these edge vertices are likely to be outside the true obstacle's boundary, their inclusion could cause the graph to misrepresent the obstacle's location and shape.

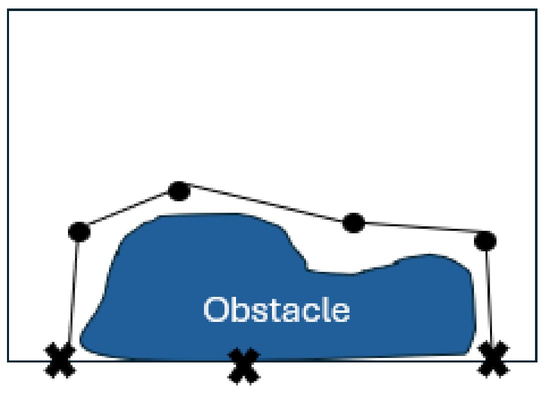


Figure 3.3: Vertices of the polygon located close to the periphery of the captured image are disregarded

The primary concern with incorporating such edge vertices is that it may lead to a situation where the ROV's path planning algorithm does not accurately avoid the obstacle. Specifically, if the edges associated with these vertices extend beyond the actual edge of the image, the ROV might be directed towards areas where the obstacle is actually present but not captured. Consequently, this could result in potential collisions between the ROV and the obstacle, undermining the effectiveness of the obstacle avoidance strategy. Therefore, to ensure a safe and accurate navigation, these peripheral vertices are excluded from the graph to prevent any misleading conclusions about the obstacle's extent and to maintain a reliable path planning system.

3. Upon detecting an obstacle, the ROV's path is no longer aligned with the originally planned route. Consequently, a graph search algorithm is employed to determine a new path around the obstacle. The search begins at the point where the ROV strays from the pre-planned trajectory, establishing this point as the start node in the graph for recalculating the path.

4. Once the ROV has successfully navigated around the obstacle and no further obstacles are detected, it resumes its journey towards the subsequent waypoint as defined in the original path plan.

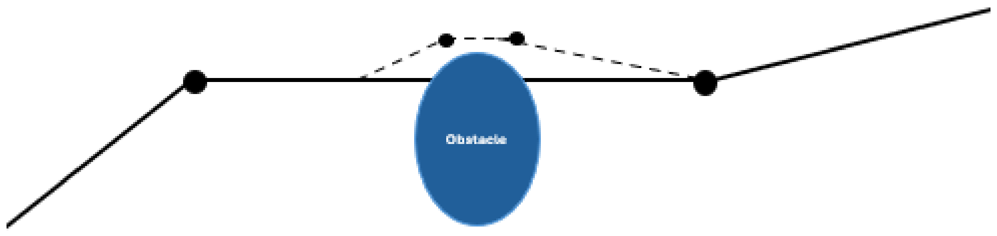


Figure 3.4: Obstacle avoidance process

3.2 Dijkstra's Algorithm

Dijkstra's algorithm is a fundamental algorithm in computer science, named after Dutch computer scientist Edsger W. Dijkstra, who first published it in 1959. It is used to find the shortest path from a starting node (often referred to as the "source" node) to all other nodes in a weighted graph.

Dijkstra's algorithm has several notable characteristics that make it particularly effective in solving specific types of problems. The algorithm employs a greedy approach, meaning it makes the optimal choice at each step with the goal of finding the global optimum. This characteristic ensures that once the shortest path to a node is identified, it remains unchanged, contributing to the algorithm's overall efficiency.

Another critical aspect of Dijkstra's algorithm is its requirement for non-negative edge weights. The algorithm assumes that once a path to a node has been established with a certain cost, no shorter path will be discovered later. Negative weights would undermine this assumption, potentially leading to incorrect results and making the algorithm unsuitable for graphs with such weights.

The time complexity of Dijkstra's algorithm varies depending on the data structures used. In its simplest form, when implemented with arrays, the algorithm has a time complexity of $O(V^2)$, where V represents the number of nodes. However, by utilizing more advanced data structures like Fibonacci heaps, the time complexity can be reduced to $O(V \log V + E)$, where E denotes the number of edges. This improvement makes the algorithm more suitable for larger graphs.

Dijkstra's algorithm has a wide range of applications across various fields. In computer networks, it plays a crucial role in network routing protocols, determining the shortest path for data to travel across routers. In Geographic Information Systems (GIS), the algorithm is integral to mapping software, helping to find the shortest path between locations, such as in GPS navigation systems. The algorithm is also widely used in robotics and artificial intelligence for path-finding, enabling robots to navigate through environments efficiently. Additionally, in telecommunications, Dijkstra's algorithm is employed to optimize the routing of signals across complex networks, ensuring efficient communication pathways [3].

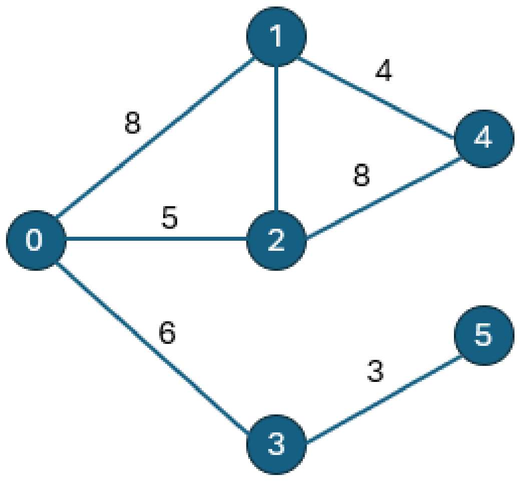
Dijkstra's Algorithm steps:

1. Set the distance to all nodes as ∞ except for the starting node, which is set to 0.
2. Mark all nodes as non-visited, including the starting node.
3. While there are non-visited nodes:
   (a) Set the non-visited node with the smallest current distance as the current node C.
   (b) For each neighbor N of the current node C:
      i. Calculate the potential new distance through C as:

      $$\text{New Distance} = \text{Current Distance of } C + \text{Weight of Edge } C\text{–}N$$

      ii. If this new distance is smaller than the current distance of N, update the distance of N.
   (c) Mark the current node C as visited.
4. Repeat the process from step 3 until the destination node is marked as visited.

Example 3.2.1 (Finding the Shortest Path using Dijkstra's Algorithm)**.** To illustrate how Dijkstra's Algorithm functions, let's consider the following example.



To find the shortest path from the start vertex (0) to all other vertices in the provided weighted graph using Dijkstra's Algorithm, we perform the following steps.

First, initialize the distance table by setting the distance to all nodes as ∞, except for the starting node (vertex 0), which is set to 0. This gives the initial distances as follows:
Distance to 0: 0,
Distance to 1: ∞,
Distance to 2: ∞,
Distance to 3: ∞,
Distance to 4: ∞,
Distance to 5: ∞.
All nodes are initially marked as non-visited.

We then proceed by iterating through the non-visited nodes, selecting the node with the smallest current distance as the current node and updating the distances to its neighbors. The process continues until all nodes are visited.

In the first iteration, the current node is vertex 0 because it has the smallest distance (0). We update the distances of its neighbors (vertices 1, 2, and 3). The new distance to vertex 1 is calculated as $0 + 8 = 8$, to vertex 2 as $0 + 5 = 5$, and to vertex 3 as $0 + 6 = 6$. The updated distances after this iteration are:

Distance to 1: 8, Distance to 2: 5, Distance to 3: 6.

Vertex 0 is then marked as visited.

In the second iteration, vertex 2 becomes the current node as it has the smallest distance among non-visited nodes (5). We then update the distances of its neighbors (vertices 0, 1, 3, and 4). The new distance to vertex 4 is calculated as $5+8=13$. Since the current distances to vertices 1 and 3 are smaller than their potential new distances, they are not updated. The updated distance to vertex 4 becomes 13:

Distance to 4: 13.

Vertex 2 is marked as visited.

In the third iteration, vertex 3 is chosen as the current node, having the smallest distance among non-visited nodes (6). The distance to its neighbor, vertex 5, is updated to $6+3=9$. The updated distances are:

Distance to 5: 9.

Vertex 3 is marked as visited.

Next, vertex 1 is selected as the current node because it has the smallest distance among non-visited nodes (8). We update the distance to its neighbor, vertex 4, to $8+4=12$, which is smaller than the current distance of 13. Thus, the distance to vertex 4 is updated:

Distance to 4: 12.

Vertex 1 is marked as visited.

In the fifth iteration, vertex 5 is the current node with a distance of 9. However, all its neighbors are already visited, so no further updates are necessary. Vertex 5 is marked as visited.

Finally, in the sixth iteration, vertex 4 is selected as the current node with a distance of 12. Again, all its neighbors are already visited, so no further updates are needed. Vertex 4 is marked as visited.

At this point, all nodes have been visited, and the shortest distances from vertex 0 to all other vertices have been determined.

The final shortest distances from vertex 0 to all other vertices are:
Distance to 0: 0,
Distance to 1: 8,
Distance to 2: 5,
Distance to 3: 6,
Distance to 4: 12,
Distance to 5: 9.

3.3 Implementation of Dijkstra Algorithm to Obstacle Avoidance Problem

In the context of real-time underwater navigation, efficiently avoiding obstacles is crucial for the safe operation of ROVs. The dynamic nature of the underwater environment demands a robust algorithm capable of recalculating paths as new obstacles are detected. Dijkstra's algorithm, renowned for its effectiveness in finding the shortest paths in a graph, is particularly well-suited for this task. By integrating Dijkstra's algorithm with the ROV's obstacle detection system, we can ensure that the vehicle can dynamically adjust its path, avoiding obstacles while minimizing detours. The following section details the implementation of Dijkstra's algorithm specifically tailored to address the obstacle avoidance problem in a 3D underwater environment.

The first step in the obstacle avoidance process involves updating the graph that represents the ROV's environment. As described in problem definition, this graph consists of vertices corresponding to points in 3D space and edges representing the Euclidean distances between these points. The vertices are updated dynamically based on the images captured by the ROV, which are processed to detect obstacles. When a new set of points is identified, representing the vertices of an obstacle, the graph needs to be updated to incorporate these points.

The `updateGraph` algorithm begins by connecting all the vertices of the newly detected obstacle to each other. This is done by iterating through the list of new vertices and calculating the Euclidean distances between every pair of vertices, thereby adding the corresponding edges to the graph. The graph is undirected, meaning that the distance from point u to point v is the same as the distance from v to u, which is why the distance is stored symmetrically in the graph matrix.

Vertices from the current update are connected in a way that every vertex, except for the last one, is connected to the subsequent one and every vertex, except the first one, is connected to the previous one. Finally, first and the last vertex are connected. After connecting the vertices of the current update, the algorithm proceeds to connect these new vertices with the vertices of the previous update. This ensures that the graph remains fully connected, accounting for all obstacles that the ROV has encountered so far. By continually updating the graph in this manner, the algorithm maintains an accurate and up-to-date representation of the ROV's environment, which is crucial for the pathfinding process.

Algorithm 1 Update Graph

```
Require: graph matrix of size MAX_POINTS × MAX_POINTS
Require: points array of all points
Require: setSize array of input sizes
Require: inputNumber number of updates
Require: totalPoints size of array points
 1: for u ← totalPoints - setSize[inputNumber] +1 to totalPoints do    ▷ Connect all nodes in the current update with each other
 2:     if u > then totalPoints - setSize[inputNumber] +1    ▷ Connect to the previous node (if not the first node)
 3:         graph[u][u-1] ← euclideanDistance(points[u], points[u-1])
 4:         graph[u-1][u] ← graph[u][u-1]    ▷ Graph is undirected
 5:     end if
 6:     if u < totalPoints then    ▷ Connect to the following node (if not the last node)
 7:
 8:         graph[u][u+1] ← euclideanDistance(points[u], points[u+1])
 9:         graph[u+1][u] ← graph[u][u+1]    ▷ Graph is undirected
10:     end if
11: end for
12: for u ← totalPoints - setSize[inputNumber] - setSize[inputNumber - 1] to totalPoints - setSize[inputNumber] do
13:     for v ← totalPoints - setSize[inputNumber] +1 to totalPoints do
14:         graph[u][v] ← euclideanDistance(points[u], points[v])    ▷ Connect all nodes in the current update with previous update
15:         graph[v][u] ← graph[u][v]    ▷ Graph is undirected
16:     end for
17: end for
```

Once the graph is updated with the new obstacle information, the next step is to compute the shortest path from the ROV's current location to its next target, bypassing any detected obstacles. This is accomplished using Dijkstra's algorithm, which is well-suited for finding the shortest path in a weighted graph where the weights represent distances.

The algorithm starts by initializing the distance to all nodes in the graph as infinite, except for the starting node, which is set to zero. This initialization reflects the fact that initially, the shortest path to any node is unknown, except for the starting node itself. A set called `sptSet` (Shortest Path Tree Set) is used to keep track of the nodes that have been processed, ensuring that each node is only processed once.

The core of Dijkstra's algorithm is its greedy approach: at each step, the algorithm selects the unprocessed node with the smallest known distance from the starting node and explores its neighbors. For each neighbor, the algorithm calculates the potential new distance by adding the distance from the current node to the weight of the edge connecting the current node to the neighbor. If this new distance is shorter than the currently known distance to the neighbor, the algorithm updates the neighbor's distance and records the current node as its predecessor.

The process repeats until all nodes have been processed or until the destination node is reached. The result is a list of distances from the starting node to all other nodes in the graph, with the shortest path to each node being determined by following the recorded predecessors back from the destination node to the starting node.

Algorithm 2 Dijkstra Algorithm

```
Require: graph matrix of size MAX_POINTS $\times$ MAX_POINTS
Require: totalPoints number of nodes in the graph
Require: currentLocation source node
Require: inputSet array of goal nodes
 1: Initialize distances as array of size totalPoints with all elements set
    to $\infty$
 2: distances[currentLocation] $\leftarrow 0$    ▷ Distance to the source is zero
 3: Initialize sptSet as array of size totalPoints with all elements set to
    false
 4: Initialize previous as array of size totalPoints with all elements set
    to null
 5: for count $\leftarrow 0$ to totalPoints - 1 do
 6:     u $\leftarrow$ vertex with minimum dist not in sptSet
 7:     sptSet[u] $\leftarrow$ true                    ▷ Mark vertex u as processed
 8:     if u is in inputSet then
 9:         break                           ▷ Stop if we reach a goal node
10:     end if
11:     for v $\leftarrow 0$ to totalPoints - 1 do
12:         if not sptSet[v] and graph[u][v] $\neq 0$ and distances[u]
    $\neq \infty$ and distances[u] + graph[u][v] $<$ distances[v] then
13:             distances[v] $\leftarrow$ distances[u] + graph[u][v]
14:             previous[v] $\leftarrow u$                ▷ Track the predecessor of v
15:         end if
16:     end for
17: end for
18: Initialize goalNode as node in inputSet with the minimum distances
19: Initialize path as an empty list
20: currentNode $\leftarrow$ goalNode             ▷ Start with the goal node found
21: while currentNode is not null do
22:     insert(currentNode, path)          ▷ Insert currentNode at the
    beginning of the path
23:     currentNode $\leftarrow$ previous[currentNode]
24: end while
25: return path ▷ Return the list of nodes forming the shortest path to a
    goal node
```

In a dynamic environment, where the ROV continually encounters new obstacles, the graph and shortest path need to be updated regularly. The `graph-update` algorithm combines the graph updating and shortest path calculation into a single process, allowing the ROV to adjust its path in real-time as it navigates through the underwater environment.

The algorithm begins by initializing the graph with the ROV's current location and then enters a loop where it continuously reads input from the ROV's sensors. Each time a new obstacle is detected, the graph is updated with the new obstacle points, and Dijkstra's algorithm is invoked to recalculate the shortest path from the ROV's current location to the next target. The newly calculated path is then used to guide the ROV around the obstacle.

If no new obstacles are detected, the algorithm reinitializes its data structures, preparing for the next iteration. This reinitialization is crucial because it ensures that the algorithm does not retain outdated information from previous iterations, which could lead to incorrect path calculations.

By dynamically updating the graph and recalculating the shortest path, the algorithm ensures that the ROV can adapt to changes in its environment and navigate around obstacles in an efficient manner. This approach leverages the strengths of Dijkstra's algorithm in finding the shortest path while also accounting for the dynamic nature of the underwater environment, where obstacles can appear unexpectedly at any time.

Algorithm 3 Dynamic Graph Update and Shortest Path Calculation

```
Require: Initial 3D point currentLocation    ▷ Point where ROV
    deviates from pre-planned trajectory and a source node of the graph
Require: Array of 3D points inputSet
    Initialize points as a null array of size MAX_POINTS    ▷ Array to hold
    the points from the inputs
 2: Initialize graph as a null matrix of size MAX_POINTS × MAX_POINTS
    Initialize setSize as a null array of size MAX_POINTS    ▷ Array to hold
    the sizes of each input set
 4: Initialize inputNumber as integer    ▷ Counter for the number of
    non-empty inputs
    Initialize totalPoints as integer
 6: totalPoints ← 1
    setSize[0] ← 1
 8: inputNumber ← 0
    while True do
10:     Read inputSet
        Remove noncompliant points from inputSet
12:     Read currentLocation
        points[0] ← currentLocation
14:     if inputSet is not empty then
            inputNumber ← inputNumber + 1
16:         setSize[inputNumber] ← number of points in inputSet
            Update points with inputSet
18:         totalPoints ← totalPoints + setSize[inputNumber]
            Call updateGraph(graph, points, setSizes,
    inputNumber, totalPoints)
20:         Call dijkstra(graph, totalPoints, currentLocation)
            newPath[MAX_POINTS] ← dijkstra(graph, totalPoints,
    currentLocation)
22:         Print newPath
        end if
24:     Reinitialize all data structures    ▷ No obstacle or avoidance process
    is done
        points[0] ← currentLocation
26:     totalPoints ← 1
        setSize[0] ← 1
28:     inputNumber ← 0
        sleep(Δ t)
30: end while
```

3.3.1 Experiment

The algorithm from the section before is now tested on synthetic data, which consists of 3D point that represent current location of the ROV and multiple sets of points that mimic the vertices of the polygon that approximates the obstacle in captured pictures, as explained before.

Input:
{0.0, 0.0, 0.0}
{ ∅,
{ {1, 3, 0}, {1, -3, 0}, {1.5, 1, 3}, {1.5, -2, 2.5} },
{ {7, 6, 3.5}, {7.5, -2.7, 5.5}, {7.5, 2, 5}, {7, -6.7, -3.1} },
{ {12.1, 2, 7.3}, {12.4, 3.6, 5.5}, {13, -4.6, 5.4} },
∅}

Here we label input points and visualize the data and graph.
O=(0.0,0.0,0.0),
A=(1,3,0),
B=(1,-3,0),
C=(1.5,1,3),
D=(1.5,-2,2.5),
F=(7,6,3.5),
G=(7.5,-2.7,5.5),
H=(7.5,2,5),
J=(7,-6.7,-3.1),
K=(12.1,2,7.3),
L=(13, -4.6, 5.4),
I=(12.4, 3.6, 5.5).

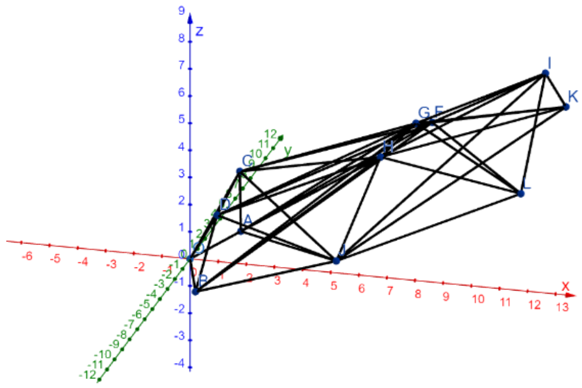


Figure 3.5: Data represented in 3D space

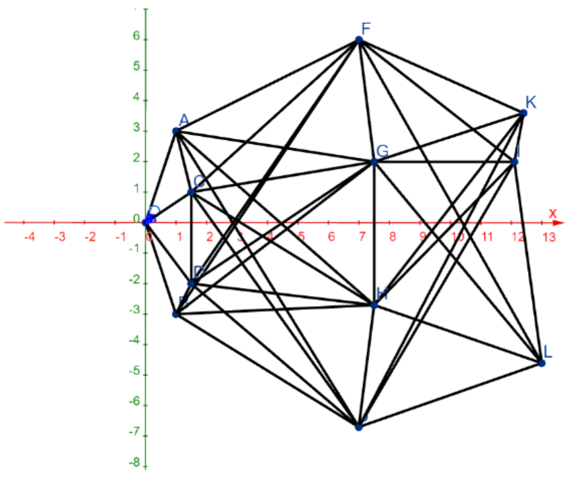


Figure 3.6: Floor plan of data

Output:

```
(0.00, 0.00, 0.00)
(1.00, 3.00, 0.00)
(7.00, 6.00, 3.50)
(12.10, 2.00, 7.30)
Time taken: 0.000058 seconds
```

Chapter 4

Conclusion

In summary, the integration of optimization techniques into the operational framework of Remotely Operated Vehicles (ROVs) is essential for enhancing their effectiveness and safety in complex underwater environments. As ROVs continue to play a vital role in various industries, the development of sophisticated optimization models that address multi-objective challenges and obstacle avoidance strategies will be crucial for their future success.

The ability to navigate dynamically changing environments while balancing multiple objectives is a testament to the power of optimization in real-world applications. By employing methods such as the weighted sum approach, decision-makers can tailor ROV operations to meet specific mission goals, ensuring that these vehicles can adapt to unforeseen challenges while maximizing their operational efficiency.

Moreover, the implementation of advanced obstacle avoidance algorithms, such as those based on Dijkstra's Algorithm, highlights the importance of real-time data processing and decision-making in the field of robotics. As technology continues to advance, the potential for ROVs to operate autonomously and efficiently in complex underwater scenarios will only increase, paving the way for new discoveries and innovations.

Ultimately, the ongoing research and development in optimization techniques for ROVs not only enhance their operational capabilities but also contribute to the broader understanding of marine environments. By ensuring that ROVs can navigate safely and effectively, we can unlock new opportunities for exploration, conservation, and resource management, thereby making a lasting impact on our understanding of the underwater world.

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